MA 162: Finite Mathematics - Sections 2.4/2.5

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Announcements:

- Homework 2.3 due Friday at 6pm.
- Homework 2.4/2.5 due next Tuesday at 6pm.
- Exam #1 is Monday 5-7pm (see website for room locations)

2.4 - Matrices

- A matrix is a rectangular array of numbers.
- The size (dimension) of a matrix is given by two numbers: the first is the number of rows in the matrix; the second is the number of columns of the matrix.
- For example,

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \\ 4 & 4 \end{bmatrix}$$

  is a 4×2 matrix.
- A square matrix is a matrix with the same number of rows and columns.
- The entry in the ith row and jth column of a matrix named A is denoted a_{ij}.

2.4 - Equality of Matrices

- Two matrices are equal when they have the same dimensions and the same entry in every position.
- For example,

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \neq \begin{bmatrix} 1 & 2 & 6 \\ 4 & 5 & 3 \end{bmatrix}$$

  because they differ in the $(1, 3)$ and $(2, 3)$ entries.
- Also,

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \neq \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

  because they have different dimensions.

2.4 - Equality of Matrices

- Find the values of u, x, y, and z so that

$$\begin{bmatrix} 2x\text{-}2 & 3 & 2 \\ 2 & 4 & y\text{-}2 \\ 2z & \text{-}3 & 2 \end{bmatrix} = \begin{bmatrix} 3 & u & 2 \\ 2 & 4 & 5 \\ 4 & \text{-}2 & 2 \end{bmatrix}$$

2.4 - Addition and Subtraction of Matrices

- Matrices can only be added or subtracted if they have the same dimensions.
- Addition and subtraction are done componentwise.

- $\begin{bmatrix} 2 & -1 & 3 \\ 9 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & -1 \\ -6 & 4 & 2 \end{bmatrix} =$

- $\begin{bmatrix} 6 & 3 & 8 \\ 4 & 5 & 6 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -1 \\ 0 & -5 & -7 \end{bmatrix} =$

- For addition, $A + B = B + A$ and $(A + B) + C = A + (B + C)$.

2.4 - Scalar Multiplication of Matrices

- If A is a matrix and c is a real number, then the scalar product cA is the matrix obtained by multiplying each entry of A by the number c.

- For example,

$$3 \cdot \begin{bmatrix} 2 & 3 & \text{-}1 \\ 4 & 5 & \text{-}2 \\ 1 & \text{-}3 & 6 \end{bmatrix} =$$

- The size of the matrix does not matter here. You can multiply a number times any size matrix in this fashion.

- For scalar multiplication, $c(A + B) = cA + cB$.

2.4 - Application question

- Ashley holds stock in four different companies: Google, Ebay, Priceline, and Netflix. She has 100 shares of Google, 175 shares of Ebay, 200 shares of Priceline, and 30 shares of Netflix at the beginning of the year. Over the course of the year, Ashley buys 100, 125, 40, and 90 shares of each company respectively. Write a matrix equation whose result gives the total number of shares Ashley has of each company at the end of the year.

2.5 - Tan, Problem 49

- Ashley's stock holdings are given by a matrix A

$$A = [\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad]$$

At the close of trading on Monday, Tuesday, and Wednesday of a certain week, the prices (in dollars) of the stocks were given by matrix B:

	Mon.	Tues.	Wed.
Google	821.50	838.60	831.38
Ebay	55.48	55.26	53.37
Priceline	714.01	718.41	718.90
Netflix	181.21	181.73	182.94

Use matrix multiplication to find a matrix C giving the total value of Ashley's stock holdings on each of the three days.

2.5 - Matrix Multiplication

- Matrix mulitplication can only be done in some cases. The simplest case multiplies a matrix with only one row times a matrix with only one column.
- $\begin{bmatrix} 2 & -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \\ -2 \end{bmatrix} =$
- In general, matrix multiplication can be done when the number of columns in the first matrix equals the number of rows in the second matrix.

2.5 - Matrix Multiplication

- For larger matrices, do the same process for each possible pair using a row from the first matrix and column of the second matrix. For example, to do

$$\begin{bmatrix} 2 & \text{-1} & 3 \\ 9 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \text{-1} & 2 \\ \text{-6} & 4 \\ 5 & 8 \end{bmatrix}$$

you would first start by doing

$$\begin{bmatrix} 2 & \text{-1} & 3 \end{bmatrix} \cdot \begin{bmatrix} \text{-1} \\ \text{-6} \\ 5 \end{bmatrix} = [2(-1) + (-1)(-6) + 3(5)] = [19]$$

2.5 - Matrix Multiplication

- Similarly,

$$\begin{bmatrix} 2 & \text{-1} & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix} = [2(2) + (-1)(4) + 3(8)] = [24]$$

$$\begin{bmatrix} 9 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \text{-1} \\ \text{-6} \\ 5 \end{bmatrix} = [9(-1) + 2(-6) + 1(5)] = [-16]$$

$$\begin{bmatrix} 9 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix} = [9(2) + 2(4) + 1(8)] = [34]$$

- So, the final result is

$$\begin{bmatrix} 2 & \text{-1} & 3 \\ 9 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \text{-1} & 2 \\ \text{-6} & 4 \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} 19 & 24 \\ \text{-16} & 34 \end{bmatrix}$$

2.5 - Matrix Multiplication

Here are some other important things about matrix multiplication:

- $(AB)C = A(BC)$ (associative law)
- $A(B + C) = AB + AC$ (distributive law)
- $AB \neq BA$ (matrix multiplication is not necessarily commutative)

There is a special matrix that shows up (especially in section 2.6) called an identity matrix.

- An identity matrix is a square matrix with 1s along the diagonal and 0s everywhere off the diagonal. For example, the 3×3 identity matrix looks like

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2.5 - Matrix Representation

- Consider the system of equations

$$\begin{array}{rcrcrcr} x & + & y & - & 2z & = & 0 \\ 2x & - & 3y & + & 3z & = & 2 \\ x & + & y & - & 2z & = & 1 \end{array}$$

- Let

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & -3 & 3 \\ 1 & 1 & -2 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} B = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

- Then the system of equations can be represented by the matrix equation

$$AX = B$$