Worksheet 3

Problem 1

a) $x + 4y = 7 \implies y = \frac{7-x}{4} = \frac{7}{4} - \frac{x}{4}$

$\frac{x}{2} + 2y = 5 \implies y = \frac{5 - \frac{x}{2}}{2} = \frac{5}{2} - \frac{x}{4}$

both lines have the same slope, but different y intercepts.
This implies that the lines are parallel & distinct. Hence there are
no solutions

b) $x + 2y = 7 \implies y = \frac{7-x}{2} = \frac{7}{2} - \frac{x}{2}$

$2x - y = 4 \implies y = 2x - 4$

Set the two equations equal to one another

$$\frac{7}{2} - \frac{x}{2} = 2x - 4$$
$$7 - x = 4x - 8$$
$$15 = 5x$$
$$3 = x$$

Evaluate the function at x to obtain the y-coordinate.

$y(3) = 2(3) - 4 = 6 - 4 = 2$. So the $\boxed{\text{unique solution is } (3,2)}$

c) $2x - 5y = 10 \implies y = \frac{2x-10}{5} = \frac{2}{5}x - 2$

$6x - 15y = 30 \implies y = \frac{6x-30}{15} = \frac{2}{5}x - 2$

Both lines are parallel and coincident since they have the same slope & y-intercept. The solutions are given by $\boxed{(t, \frac{2t}{5} - 2)}$

d) $4x - 5y = 14 \implies y = \frac{4x-14}{5} = \frac{4}{5}x - \frac{14}{5}$

$2x + 3y = -4 \implies y = \frac{-(4+2x)}{3} = -\frac{2}{3}x - \frac{4}{3}$

Set the two equations equal to one another

$$\frac{4}{5}x - \frac{14}{5} = -\frac{2}{3}x - \frac{4}{3}$$

$$4x - 14 = -\frac{10}{3}x - \frac{20}{3}$$

$$12x - 42 = -10x - 20$$

$$22x = 22$$

$$x = 1$$

Evaluate the function at x to obtain the y-coordinate

$$y(1) = -\frac{2}{3}(1) - \frac{4}{3} = -\frac{2}{3} - \frac{4}{3} = -\frac{6}{3} = -2$$

$\boxed{\text{The uniqe solution is } (1,-2)}$

Problem 2

A linear system of two equations has no solutions if the lines are parallel & concident.

$$2x - y = 3 \Longrightarrow y = 2x - 3$$

$$4x + Ky = 4 \Longrightarrow y = \frac{4 - 4x}{K} = \frac{4}{K} - \frac{4}{K}x$$

Set the slopes equal to one another to ensure they are parallel

$$2 = \frac{-4}{K}$$

$$-2K = 4$$

$$K = -2$$

Since the y-intercepts of both lines are different, then system of equations have no solution when $\boxed{K=-2}$

Problem 3

Let X denote the number of children. And let Y denote the number of adult passengers. We're given information about the total number of passengers and the total revenue generated. We need to relate X, Y to these quantities.

Total # of passengers = 1000 = X + Y

Total Revenue = (cost/child)(# of children) + (cost/adult)(# of adults)

$$1300 = .5X + 1.5Y$$

So our system of equations are

$$\boxed{\begin{aligned} 1000 &= X + Y \\ 1300 &= .5X + 1.5Y \end{aligned}}$$

Problem 4

Let X denote the land alloted to corn & Y denote the land alloted to wheat. We know the total acres of land & amount of cash availible to spend. So

Total acres of land = 500 = X + Y

Total Expences = (cost to produce corn / acre)(# of acres of corn) + (cost to produce wheat / acre)(# of acres of wheat)

$$18,600 = 42X + 30Y$$

So our system of equations are

$$\boxed{\begin{aligned} 500 &= X + Y \\ 18,600 &= 42X + 30Y \end{aligned}}$$

Problem 5

Let X denote the number of front orchestra seats, Y denote the number of rear orchestra seats, and Z denote the number of front balcony seats. Following the same procedure as problem 4 & Problem 5

Total # of seats $= 1000 = X + Y + Z$

Total Receipts $= 62{,}800 = 80X + 60Y + 50Z$

The last equation will require more work. The "combined number of tickets sold for the front orchestra & rear orchestra" is equivalent to $X + Z$. "Exceed twice the number of front balcony tickets sold by 400" implies that

$(X+Z) - Y = 400$. All in all

$$\boxed{\begin{aligned} 1000 &= X + Y + Z \\ 62{,}800 &= 80X + 60Y + 50Z \\ 400 &= X - Y - Z \end{aligned}}$$