

$$1) a) \begin{bmatrix} 3 & 3 & | & 6 \\ 2 & 1 & | & 4 \end{bmatrix} \xrightarrow{\frac{1}{3}R_1} \begin{bmatrix} 1 & 1 & | & 2 \\ 2 & 1 & | & 4 \end{bmatrix} \xrightarrow{R_2 - 2R_1} \begin{bmatrix} 1 & 1 & | & 2 \\ 0 & -1 & | & 0 \end{bmatrix}$$

$$\xrightarrow{-\frac{1}{5}R_2} \begin{bmatrix} 1 & 3 & | & 2 \\ 0 & 1 & | & 0 \end{bmatrix} \xrightarrow{R_1 - 3R_2} \begin{bmatrix} 1 & 0 & | & 2 \\ 0 & 1 & | & 0 \end{bmatrix}$$

$$b) \begin{bmatrix} 1 & 2 & | & 1 \\ 2 & 3 & | & -1 \end{bmatrix} \xrightarrow{R_2 - 2R_1} \begin{bmatrix} 1 & 2 & | & 1 \\ 0 & -1 & | & -3 \end{bmatrix} \xrightarrow{-R_2} \begin{bmatrix} 1 & 2 & | & 1 \\ 0 & 1 & | & 3 \end{bmatrix} \xrightarrow{R_1 - 2R_2} \begin{bmatrix} 1 & 0 & | & -5 \\ 0 & 1 & | & 3 \end{bmatrix}$$

$$2) a) \begin{bmatrix} 2 & 1 & -2 & | & 4 \\ 1 & 3 & -1 & | & -3 \\ 3 & 4 & -1 & | & 7 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{bmatrix} 1 & 1/2 & -1 & | & 2 \\ 1 & 3 & -1 & | & -3 \\ 3 & 4 & -1 & | & 7 \end{bmatrix} \xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 1/2 & -1 & | & 2 \\ 0 & 5/2 & 0 & | & -5 \\ 0 & 5/2 & 2 & | & 1 \end{bmatrix}$$

$$\xrightarrow{\frac{2}{5}R_2} \begin{bmatrix} 1 & 1/2 & -1 & | & 2 \\ 0 & 1 & 0 & | & -2 \\ 0 & 5/2 & 2 & | & 1 \end{bmatrix} \xrightarrow{\begin{matrix} R_1 - \frac{1}{2}R_2 \\ R_3 - \frac{5}{2}R_2 \end{matrix}} \begin{bmatrix} 1 & 0 & -1 & | & 3 \\ 0 & 1 & 0 & | & -2 \\ 0 & 0 & 2 & | & 6 \end{bmatrix} \xrightarrow{\begin{matrix} \frac{1}{2}R_3 \\ R_1 + R_3 \end{matrix}} \begin{bmatrix} 1 & 0 & 0 & | & 6 \\ 0 & 1 & 0 & | & -2 \\ 0 & 0 & 1 & | & 3 \end{bmatrix}$$

This system of linear equations has a unique solution  
 $(6, -2, 3)$

$$b) \begin{bmatrix} 2 & 3 & -2 & | & 10 \\ 3 & -2 & 2 & | & 0 \\ 4 & -1 & 3 & | & -1 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{bmatrix} 1 & 3/2 & -1 & | & 5 \\ 3 & -2 & 2 & | & 0 \\ 4 & -1 & 3 & | & -1 \end{bmatrix}$$

$$\xrightarrow{\begin{matrix} R_2 - 3R_1 \\ R_3 - 4R_1 \end{matrix}} \begin{bmatrix} 1 & 3/2 & -1 & | & 5 \\ 0 & -13/2 & 5 & | & -30/2 \\ 0 & -7 & 7 & | & -21 \end{bmatrix} \xrightarrow{\frac{2}{13}R_2} \begin{bmatrix} 1 & 3/2 & -1 & | & 5 \\ 0 & 1 & -10/13 & | & 30/13 \\ 0 & -7 & 7 & | & -21 \end{bmatrix}$$

$$\xrightarrow{\begin{matrix} R_1 - \frac{3}{2}R_2 \\ R_3 + 7R_2 \end{matrix}} \begin{bmatrix} 1 & 0 & 2/13 & | & 20/13 \\ 0 & 1 & -10/13 & | & 30/13 \\ 0 & 0 & 21/13 & | & -63/13 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & | & 60/21 \\ 0 & 1 & 0 & | & -100/21 \\ 0 & 0 & 1 & | & -133/21 \end{bmatrix}$$

3

$$\left[ \begin{array}{ccc|c} 3 & -2 & 4 & 12 \\ -9 & 6 & -12 & k \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{-3R_1} \left[ \begin{array}{ccc|c} -9 & 6 & -12 & 36 \\ -9 & 6 & -12 & k \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\xrightarrow{R_2 - R_1} \left[ \begin{array}{ccc|c} -9 & 6 & -12 & 36 \\ 0 & 0 & 0 & k - 36 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

The system has no solution as long as  $k - 36 \neq 0$ , that is,  $k \neq 36$ .

4) Let  $A = \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix}$ .

$$\begin{aligned} 2A + X &= B \Rightarrow X = B - 2A \\ &= \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix} - 2 \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} -4 & 2 \\ 0 & 6 \end{bmatrix} \\ &= \begin{bmatrix} -2 & 3 \\ -1 & 2 \end{bmatrix}. \end{aligned}$$

5) We can express the information in two different ways:

1)

	WHITE	BLACK	HISPANIC
WOMEN	82.6	80.5	81.2
MEN	70.8	73.3	84.8

2)

	WOMEN	MEN
WHITE	82.6	70.8
BLACK	80.5	73.3
HISPANIC	81.2	84.8

3

$$\left[ \begin{array}{ccc|c} 3 & -2 & 4 & 12 \\ -9 & 6 & -12 & k \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{-3R_1} \left[ \begin{array}{ccc|c} -9 & 6 & -12 & 36 \\ -9 & 6 & -12 & k \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\xrightarrow{R_2 - R_1} \left[ \begin{array}{ccc|c} -9 & 6 & -12 & 36 \\ 0 & 0 & 0 & k - 36 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

The system has no solution as long as  $k - 36 \neq 0$ , that is,  $k \neq 36$ .

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5) We can express the information in two different ways:

a)

	WHITE	BLACK	HISPANIC
WOMEN	82.6	80.5	81.2
MEN	70.8	73.9	84.8

b)

	WOMEN	MEN
WHITE	82.6	70.8
BLACK	80.5	73.9
HISPANIC	81.2	84.8