$\boxed{1}$ Let $A=\begin{bmatrix}3&1\\0&2\end{bmatrix}$ and $B=\begin{bmatrix}4&2\\-2&1\end{bmatrix}$

a) $(A+B)^2=\left(\begin{bmatrix}3&1\\0&2\end{bmatrix}+\begin{bmatrix}4&2\\-2&1\end{bmatrix}\right)^2=\left(\begin{bmatrix}7&3\\-2&3\end{bmatrix}\right)^2=\begin{bmatrix}7&3\\-2&3\end{bmatrix}\begin{bmatrix}7&3\\-2&3\end{bmatrix}=\begin{bmatrix}43&30\\-20&3\end{bmatrix}$

b) $A^2+2AB+B^2=\begin{bmatrix}3&1\\0&2\end{bmatrix}\begin{bmatrix}3&1\\0&2\end{bmatrix}+2\begin{bmatrix}3&1\\0&2\end{bmatrix}\begin{bmatrix}4&2\\-2&1\end{bmatrix}+\begin{bmatrix}4&2\\-2&1\end{bmatrix}\begin{bmatrix}4&2\\-2&1\end{bmatrix}$

$=\begin{bmatrix}9&5\\0&4\end{bmatrix}+\begin{bmatrix}20&14\\-8&4\end{bmatrix}+\begin{bmatrix}12&10\\-10&3\end{bmatrix}$

$=\begin{bmatrix}41&29\\-18&15\end{bmatrix}$

c) Immediate from a) and b).

$\boxed{2}$ Let $A=\begin{bmatrix}0&3&0\\1&0&1\\0&2&0\end{bmatrix}$, $B=\begin{bmatrix}2&4&5\\3&-1&-6\\4&3&4\end{bmatrix}$ and $C=\begin{bmatrix}4&5&6\\3&-1&-6\\2&2&3\end{bmatrix}$

a) $AB=\begin{bmatrix}0&3&0\\1&0&1\\0&2&0\end{bmatrix}\begin{bmatrix}2&4&5\\3&-1&-6\\4&3&4\end{bmatrix}=\begin{bmatrix}9&-3&-18\\6&7&9\\6&-2&-12\end{bmatrix}$

b) $AC=\begin{bmatrix}0&3&0\\1&0&1\\0&2&0\end{bmatrix}\begin{bmatrix}4&5&6\\3&-1&-6\\2&2&3\end{bmatrix}=\begin{bmatrix}9&-3&-18\\6&7&9\\6&-2&-12\end{bmatrix}$

c) Immediate from a) and b).

$\boxed{3}$ $\begin{bmatrix}a&b\\c&d\end{bmatrix}\begin{bmatrix}1&0\\-1&3\end{bmatrix}=\begin{bmatrix}-1&-3\\3&6\end{bmatrix}\Rightarrow\begin{bmatrix}a-b&3b\\c-d&3d\end{bmatrix}=\begin{bmatrix}-1&-3\\3&6\end{bmatrix}\Rightarrow$

$\begin{cases}a-b=-1\\3b=-3\\c-d=3\\3d=6\end{cases}\Rightarrow$ $\boxed{b=-1}$ and $\boxed{d=2}$

$a-b=-1\Rightarrow\boxed{a=0}$

$c-d=3\Rightarrow\boxed{c=5}$

$\boxed{4}$ Let $A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -2 & -2 & 1 \end{bmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ -2 & -2 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_3+2R_1]{R_2-2R_1} \left(\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & 0 & 0 \\ 0 & 3 & -4 & -2 & 1 & 0 \\ 0 & -4 & 7 & 2 & 0 & 1 \end{array}\right) \xrightarrow{\frac{1}{3}R_2} \left(\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -4/3 & -2/3 & 1/3 & 0 \\ 0 & -4 & 7 & 2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[R_3+4R_2]{R_1+R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 5/3 & 1/3 & 1/4 & 0 \\ 0 & 1 & -4/3 & -2/3 & 1/3 & 0 \\ 0 & 0 & 5/3 & -2/3 & 4/3 & 1 \end{array}\right) \xrightarrow{\frac{3}{5}R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 5/3 & 1/3 & 1 & 0 \\ 0 & 1 & -4/3 & -2/3 & 1/3 & 0 \\ 0 & 0 & 1 & -2/5 & 4/5 & 3/5 \end{array}\right)$$

$$\xrightarrow[R_1-\frac{5}{3}R_3]{R_2+\frac{4}{3}R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & -1 \\ 0 & 1 & 0 & -6/5 & 7/5 & 4/5 \\ 0 & 0 & 1 & -2/5 & 4/5 & 3/5 \end{array}\right)$$

Verification.

$$\begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -2 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 \\ -6/5 & 7/5 & 4/5 \\ -2/5 & 4/5 & 3/5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

$\boxed{5}$ The system can be written as $AX = B$ where

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 2 & -3 & 1 \\ 1 & -1 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$