Worksheet 7 · Solutions

Problem 1

Maximize $P = x + 3y + 4z$

Subject to
$x + 2y + z \leq 40$
$x + y + z \leq 30$
$x \geq 0, y \geq 0, z \geq 0$

$\Rightarrow$

x	y	z	u	v	P	const.
1	2	1	1	0	0	40
1	1	1	0	1	0	30
−1	−3	−4	0	0	1	0

Problem 2

a) Maximize $P = 3x + 4y$

Subject to
$x + y \leq 4$
$2x + y \leq 5$
$x \geq 0, y \geq 0$

$\Rightarrow$

x	y	u	v	P	const.	ratio
1	(1)	1	0	0	4	$4/1 = 4$
2	1	0	1	0	5	$5/1 = 5$
−3	−4	0	0	1	0	

pivot column b/c largest absolute value (among x & y)

pivot row b/c smallest ratio

$R_2 - R_1$
$R_3 + 4R_1$

x	y	u	v	P	const
1	(1)	1	0	0	4
1	0	−1	1	0	1
1	0	4	0	1	16

all entries in the last row to the left of the line are nonnegative, so STOP.

So $y = 4$, $v = 1$, $P = 16$
$x = 0$, $u = 0$

b) Maximize $P = 4x + 6y$

Subject to

$3x + y \leq 24$

$2x + y \leq 18$

$x + 3y \leq 24$

x	y	u	v	w	P	const.	ratio
3	1	1	0	0	0	24	$24/1 = 24$
2	1	0	1	0	0	18	$18/1 = 18$
1	(3)	0	0	1	0	24	$24/3 = 8$
-4	-6	0	0	0	1	0	

pivot column b/c largest absolute (among x & y) value

pivot row b/c smallest ratio

$R_3/3 \rightarrow$

x	y	u	v	w	P	const.
3	1	1	0	0	0	24
2	1	0	1	0	0	18
1/3	1	0	0	1/3	0	8
-4	-6	0	0	0	1	0

$R_1 - R_3$

$R_2 - R_3 \rightarrow$

$R_4 + 6R_3$

x	y	u	v	w	P	const
8/3	0	1	0	-1/3	0	16
5/3	0	0	1	-1/3	0	10
1/3	1	0	0	1/3	0	8
-2	0	0	0	2	1	48

$3R_1$

$3R_2 \rightarrow$

$3R_3$

x	y	u	v	w	P	const
8	0	3	0	-1	0	48
(5)	0	0	3	-1	0	30
1	3	0	0	1	0	24
-2	0	0	0	2	1	48

ratio

$48/8 = 6$

$30/5 = 6$ ← choose either as the pivot row

$24/1 = 24$

pivot column

$R_1/8$

$R_2/5 \rightarrow$

x	y	u	v	w	P	const
1	0	3/8	0	-1/8	0	6
(1)	0	0	3/5	-1/5	0	6
1	3	0	0	1	0	24
-2	0	0	0	2	1	48

$R_1 - R_2$

$R_3 - R_2 \rightarrow$

$R_4 + 2R_2$

x	y	u	v	w	P	const.
0	0	3/8	-3/5	3/40	0	0
1	0	0	3/5	-1/5	0	6
0	3	0	0	6/5	0	18
0	0	0	6/5	8/5	1	60

all entries in the last row to the left of the line are nonnegative, so we may stop.

$x = 6$

$3y = 18 \Rightarrow y = 6$

$(3/8)u = 0 \Rightarrow u = 0$

$P = 60$

$v = w = 0$

c)

x	y	z	u	v	w	P	const
②	1	1	1	0	0	0	10
3	5	1	0	1	0	0	45
2	5	1	0	0	1	0	40
-12	-10	-5	0	0	0	1	0

pivot column

ratio

$10/2 = 5$ ← pivot row $R_1/2$

$45/3 = 15$

$40/2 = 20$

x	y	z	u	v	w	P	const
1	1/2	1/2	1/2	0	0	0	5
3	5	1	0	1	0	0	45
2	5	1	0	0	1	0	40
-12	-10	-5	0	0	0	1	0

$R_2 - 3R_1$

$R_3 - 2R_1$

$R_4 + 12R_1$

x	y	z	u	v	w	P	const
1	1/2	1/2	1/2	0	0	0	5
0	7/2	-1/2	-3/2	1	0	0	30
0	④	0	-1	0	1	0	30
0	-4	1	6	0	0	1	60

pivot column

ratio

$5/(1/2) = 10$

$30/(7/2) = \frac{60}{7}$

$\frac{30}{4} = \frac{15}{2}$

$2R_2$

$2R_1$

$R_3/4$

x	y	z	u	v	w	P	const
2	4	1	1	0	0	0	10
0	7	-1	-3	1	0	0	60
0	1	0	-1/4	0	1/4	0	15/2
0	-4	1	6	0	0	1	60

$R_1 - R_3$

$R_2 - 7R_3$

$R_4 + 4R_3$

x	y	z	u	v	w	P	const
2	0	1	5/4	0	-1/4	0	5/2
0	0	0	11/4	1	-7/4	0	15/2
0	1	0	-1/4	0	1/4	0	15/2
9	0	1	5	0	1	1	90

$2x = 5/2 \Rightarrow x = 5/4$

$y = 15/2$

$v = 15/2$

$P = 90$

$z = u = w = 0$

Problem 3

	board feet	labor-hours	profit
tables	40	3	45
chairs	16	4	20
total	≤ 3200	≤ 520	

X = # of tables

Y = # of chairs

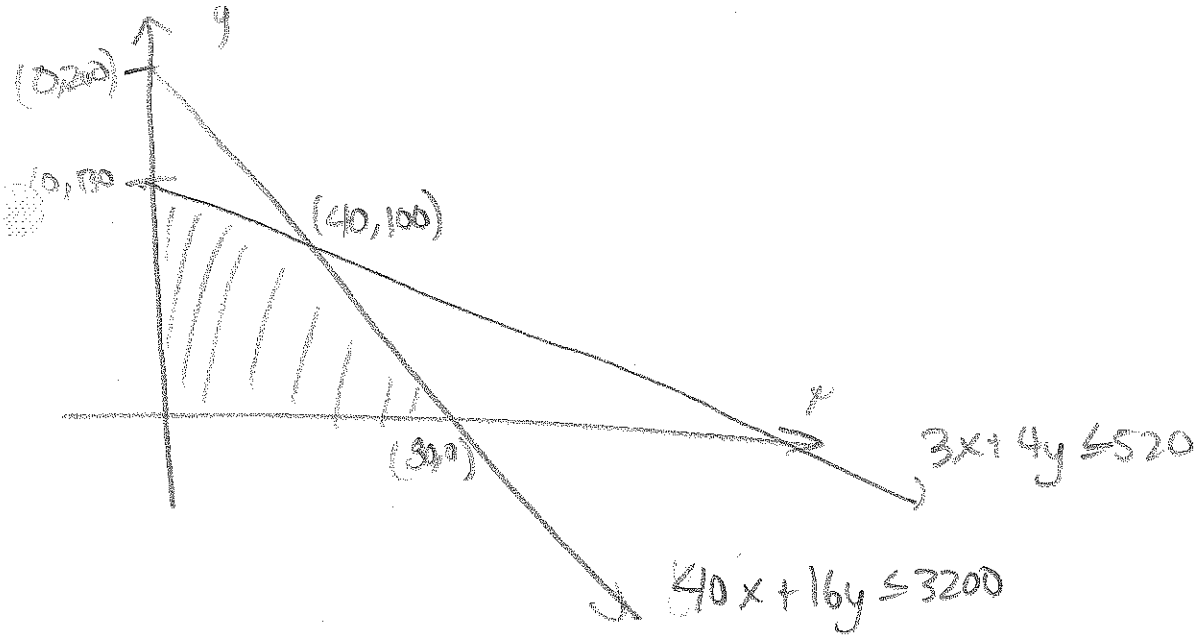
Maximize $P = 45X + 20Y$

subject to $40x + 16y \leq 3200$

$3x + 4y \leq 520$

$x \geq 0, y \geq 0$

Since there are only 2 variables, use the method of corners. In addition, the constant column vary by an order of magnitude, so fractions will be involved



The solution set has corners at (0,0), (80,0) (0,130), (40,100).
So evaluate the Profit at these four points.

x	y	P
0	0	0
80	0	3600
0	130	2600
40	100	3800

So there is a maximum profit of $3800 building 40 tables and 100 chairs

Problem 4

Let X = # of units of product A
Y = # of units of product B
Z = # of units of product C

maximize: $P = 18X + 12Y + 15Z$

subject to $2x + y + z \leq 900$

$3x + y + 2z \leq 1080$

$2x + 2y + z \leq 840$

$x \geq 0, y \geq 0, z \geq 0$

x	y	z	u	v	w	P	constant	ratio
2	1	2	1	0	0	0	900	900/2 = 450
3	1	2	0	1	0	0	1080	1080/3 = 360
2	2	1	0	0	1	0	840	840/2 = 420
-18	-12	-15	0	0	0	1	0	

$R_2/3 \rightarrow$

x	y	z	u	v	w	P	const.
2	1	1	1	0	0	0	900
1	1/3	2/3	0	1/3	0	0	360
2	2	1	0	0	1	0	840
-18	-12	-15	0	0	0	1	0

$R_1 - 2R_2 \rightarrow$

$R_3 - 2R_2$

$R_4 + 18R_2$

x	y	z	u	v	w	P	const.	ratio
0	1/3	2/3	1	-2/3	0	0	180	180/1/3 = 540
1	1/3	2/3	0	1/3	0	0	360	360/1/3 = 1080
0	4/3	-1/3	0	-2/3	1	0	120	120/4/3 = 90
0	-6	-3	0	6	0	1	6480	

$R_3/4 \rightarrow$

x	y	z	u	v	w	P	const
0	1/3	2/3	1	-2/3	0	0	180
1	1/3	2/3	0	1/3	0	0	360
0	1/3	-1/12	0	-1/6	1/4	0	30
0	-6	-3	0	6	0	1	6480

$R_1 - R_3$

$R_2 - R_3 \rightarrow$

$R_4 + 18R_3$

x	y	z	u	v	w	P	const	ratio
0	0	3/4	1	-1/2	-1/4	0	150	150/3/4 = 200
1	0	3/4	0	1/2	-1/4	0	330	330/3/4 = 440
0	1/3	-1/12	0	-1/6	1/4	0	30	30/-1/12 = 360
0	0	-9/2	0	3	9/2	1	7020	

$\frac{4}{3}R_1 \rightarrow$

x	y	z	u	v	w	P	const
0	0	(1)	$\frac{4}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	200
$\frac{9}{4}$	0	$\frac{3}{4}$	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	330
0	$\frac{1}{3}$	$-\frac{1}{12}$	0	$-\frac{1}{6}$	$\frac{1}{4}$	0	30
0	0	$-\frac{9}{2}$	0	3	$\frac{9}{2}$	1	7020

$R_2 - \frac{3}{4}R_1 \rightarrow$

$R_3 + \frac{1}{12}R_1$

$R_4 + \frac{9}{2}R_1$

x	y	z	u	v	w	P	const.
0	0	1	$\frac{4}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	200
1	0	0	-1	1	0	0	180
0	1	0	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{2}{3}$	0	140
0	0	0	6	0	3	1	7920

$x = 180$

$y = 140$

$z = 200$

$P = 7920$