1 Maximize $P = 10x + 12y$

$x + 2y \le 12$

$3x + 2y \le 24$

$x \ge 0 \quad y \ge 0$.

SIMPLEX METHOD

	x	y	u	v	P	RHS
	1	2	1	0	0	12
pivot row →	3	2	0	1	0	24
	-10	-12	0	0	1	0

↑ pivot column

$\frac{1}{3}R_2$

x	y	u	v	P	RHS
1	2	1	0	0	12
1	2/3	0	1/3	0	8
-10	-12	0	0	1	0

$R_1 - R_2$

$R_3 + 10R_2$

x	y	u	v	P	RHS
0	4/3	1	-1/3	0	4
1	2/3	0	1/3	0	8
0	-16/3	0	0	1	80

$8/(2/3) = 12 > 4/(4/3) = 3$

$\frac{3}{4}R_1$

x	y	u	v	P	RHS
0	1	3/4	-1/4	0	3
1	2/3	0	1/3	0	8
0	$-\frac{16}{3}$	0	0	1	80

$R_2 - \frac{2}{3}R_1$

$R_3 + \frac{16}{3}R_1$

x	y	u	v	P	RHS
0	1	3/4	-1/4	0	3
1	0	-1/2	1/2	0	6
0	0	4	4/3	1	96

$P = 96, \quad (3, 6)$

2 a) If the last row of the tableau is non negative then the optimal solution has been reached.

b) The pivot column is the one that contains the smallest negative element from the last row.

Once you have the pivot column, divide the RHS with the pivot column. The smallest ratio will give you the pivot row.

c) Pivot element is the intersection of pivot row with pivot column

[3]

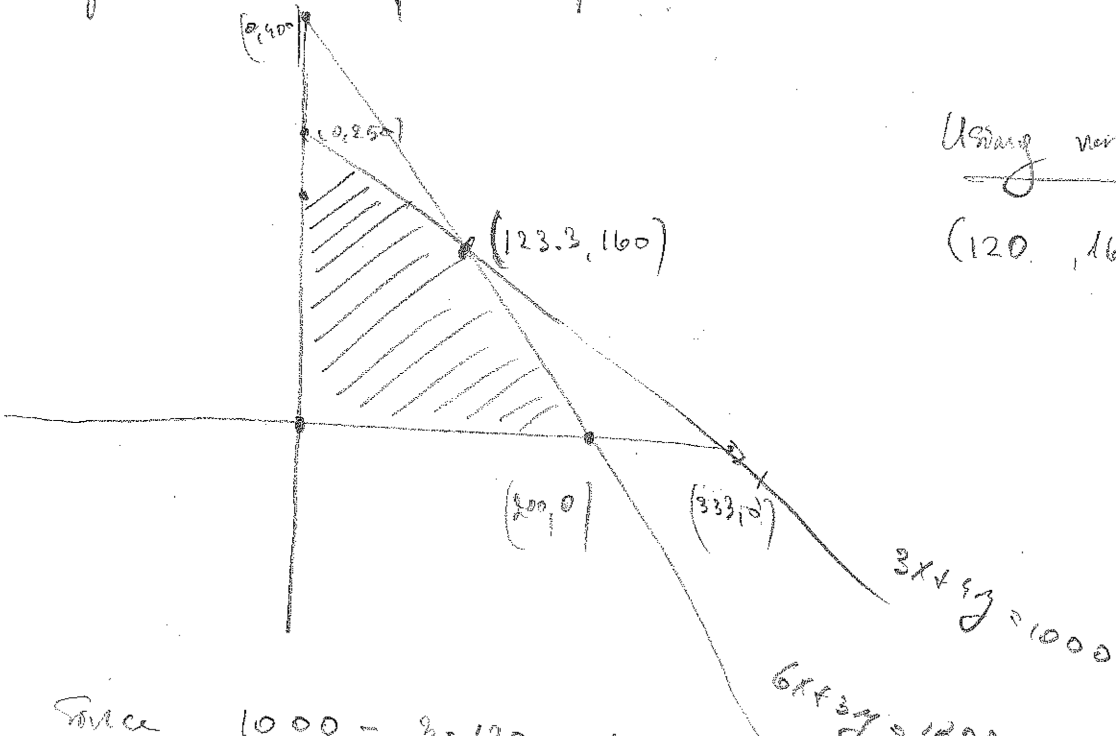
	Model A	Model B	RHS
cast iron	3	4	1000
min of labor	6	3	20·60 = 1200 min
Profit	\$2	\$1.5	

max $2x + 1.5y$

$3x + 4y \le 1000$

$6x + 3y \le 1200$

$x \ge 0, y \ge 0.$



Using vertex method

$(120, 160) \to P = 480$

Since $1000 - 3\cdot120 - 4\cdot160 = 0$, there is no cast iron left over.

Since $1200 - 6\cdot120 - 3\cdot160 = 0$, there is no labor time left over.

[4] We can rewrite the system as. $AX = C$ where

$$A = \begin{pmatrix} 1 & -2 & 4 \\ 2 & 3 & -2 \\ 1 & 4 & 6 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad C = \begin{pmatrix} 13 \\ 0 \\ -13 \end{pmatrix}$$

Find A^{-1} using Gauss-Jordan method

$$\left(\begin{array}{ccc|ccc} 1 & -2 & 4 & 1 & 0 & 0 \\ 2 & 3 & -2 & 0 & 1 & 0 \\ 1 & 4 & -6 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_3 - R_1]{R_2 - 2R_1} \left(\begin{array}{ccc|ccc} 1 & -2 & 4 & 1 & 0 & 0 \\ 0 & 7 & -10 & -2 & 1 & 0 \\ 0 & 6 & -10 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{\frac{1}{7}R_2} \left(\begin{array}{ccc|ccc} 0 & -2 & 4 & 1 & 0 & 0 \\ 0 & 1 & -10/7 & -2/7 & 1/7 & 0 \\ 0 & 6 & -10 & -1 & 0 & 1 \end{array}\right) \xrightarrow[R_3 - 6R_3]{R_1 + 2R_2} \left(\begin{array}{ccc|ccc} 0 & 0 & 8/7 & 11/7 & 2/7 & 0 \\ 0 & 1 & -10/7 & -2/7 & 1/7 & 0 \\ 0 & 0 & -10/7 & 5/7 & -6/7 & 1 \end{array}\right)$$

$$\xrightarrow[-\frac{7}{10}R_3 \quad R_1 - \frac{8}{7}R_3]{R_2 + R_3} \left(\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & -4/10 & 8/10 \\ 0 & 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & -5/10 & 6/10 & -\frac{7}{10} \end{array}\right)$$

$$X = A^{-1}B = \begin{pmatrix} 1 & -4/10 & 8/10 \\ -1 & 1 & -1 \\ -5/10 & 6/10 & -7/10 \end{pmatrix} \begin{pmatrix} 13 \\ 0 \\ -19 \end{pmatrix} = \begin{pmatrix} -11/5 \\ 6 \\ 34/5 \end{pmatrix}$$