Problem 1

a) $A^c = \{x \in U \mid x \in U \;\&\; x \notin A\} = \{2,4,6,8,10\} = B$

b) $B \cup C = \{x \in U \mid x \in B \text{ or } x \in C\} = \{1,2,4,5,6,8,9,10\}$

c) $C \cup C^c = \{x \in U \mid x \in C \text{ or } x \notin C\} = U$

d) $(A \cap B) \cup C = \{x \in U \mid x \in A \cap B \text{ or } x\, C\}$

$= \{x \in U \mid x \in A \text{ and } x \in B \text{ or } x \in C\}$

since $A \cap B = \emptyset$, so $(A \cap B) \cup C = \emptyset \cup C = C$

e) $(A \cup B \cup C)^c = \{\text{elements in } A, B, \text{or } C\}^c = \{\text{elements not in } A, B, \text{or } C\}$

Since $A \cup B \cup C = U$, $U^c = \emptyset$

f) $(A \cap B \cap C)^c = (\emptyset \cap C)^c = \emptyset^c = U$

Problem 2

a) D^c = everyone on the hospital staff who is not a Doctor

b) N^c = " " who is not a nurse

c) $N \cup D$ = " " who is a doctor or a nurse

d) $N \cap M$ = all the male nurses

e) $D \cap M$ = all the male doctors

f) $D \cap M^c = D \cap F$ = all the female doctors

Problem 3

a) $\cancel{N(A) = 15}$ $n(A) = 15$

b) $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 15 + 18 - 3 = 30$

c) $n(A^c \cap B) = 15$

d) $n(A \cap B^c) = 12$

e) $n(U) = n(A \cup B) + n([A \cup B]^c)$

$= 30 + 20 = 50$

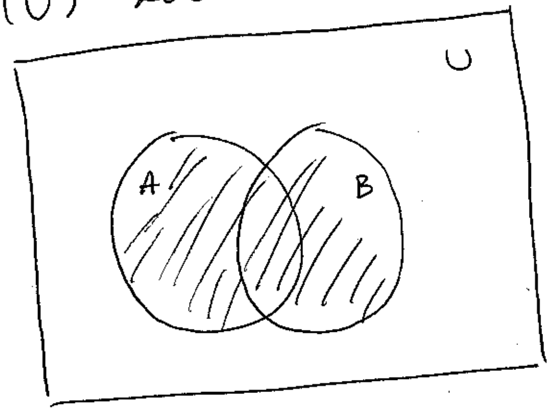
f) $n[(A \cap B)^c] = n(U) - n(A \cap B) = 47$

Problem 4

Let U = all the people polled. Let A denote the people who used discount brokers & B denote the people who used full service brokers. By assumption

$n(U) = 200$ $n(A) = 120$ $n(B) = 126$ $n(A \cap B) = 64$

a)

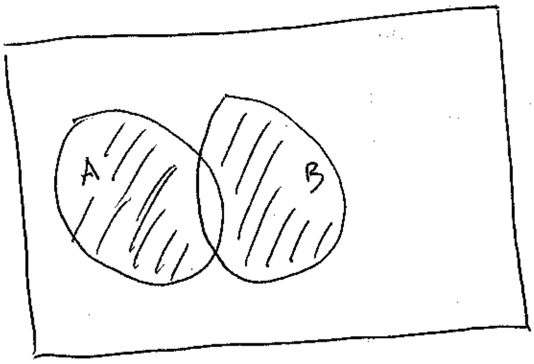


The number of people who use at least one kind of broker is given by $n(A \cup B)$.

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 120 + 126 - 64 = 182$

b)



The number of people who use only one type of broker is

$n[(A \cap B^c) \cup (B \cap A^c)]$. Since both sets are disjoint

$n([A \cap B^c] \cup [B \cap A^c]) = n(A \cap B^c) + n(B \cap A^c)$

Additionally $n(A \cap B^c) = n(A) - n(A \cap B)$ Similarly

$n(B \cap A^c) = n(B) - n(A \cap B)$.

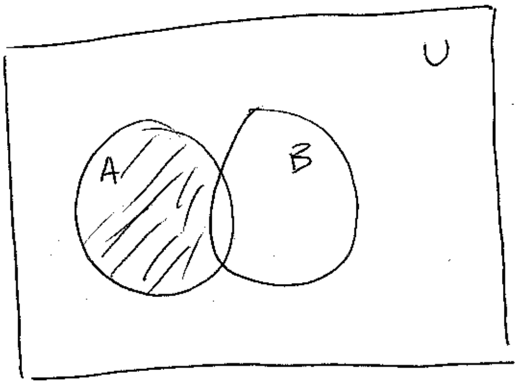
Therefore

$$n\left([A\cap B^c]\cup[B\cap A^c]\right) = n(A\cap B^c) + n(B\cap A^c)$$

$$= \{n(A) - n(A\cap B)\} + \{n(B) - n(A\cap B)\}$$

$$= 120 + 126 - 128 = 118$$

c)



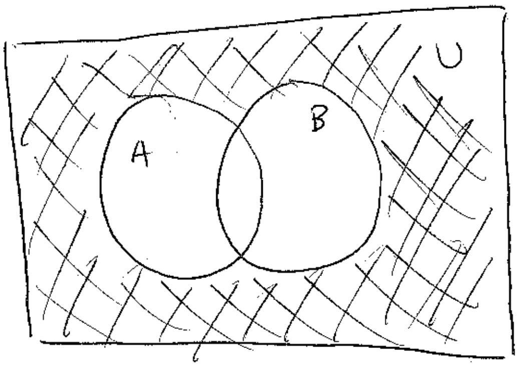
The number of people who use only discount brokers is

$$n[A\cap B^c] = n(A) - n(A\cap B)$$

$$= 120 - 64 = 56$$

(by the same procedure as the previous part)

d)



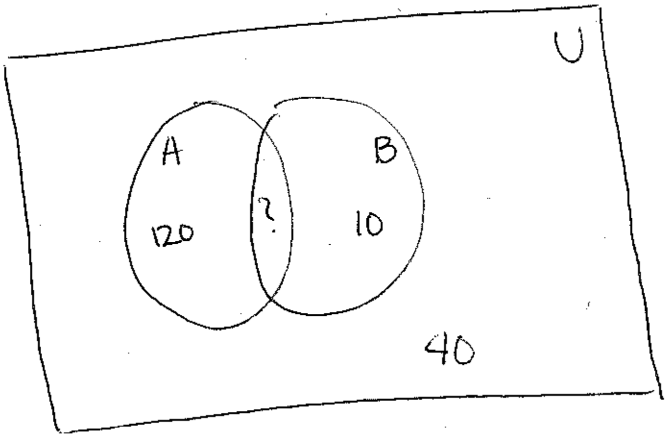
The number of people who don't use any broker is

$$n[(A\cup B)^c] = n(U) - n(A\cup B)$$

$$= 200 - 182 = 18$$

Problem 5

Let U = people surveyed, A = households who own desktops, B = households who own tablets. Filling in the picture below



we want $n(A\cap B)$. We know

$n(U) = 200$ $\quad n[A\cap B^c] = 120$

$n((A\cup B)^c) = 40$ $\quad n[B\cap A^c] = 10$

By the picture

$$U = (A \cap B^c) \cup (B \cap A^c) \cup (A \cup B)^c \cup (A \cap B)$$

All of the above sets are disjoint, so

$$n(U) = n[A \cap B^c] + n[B \cap A^c] + n[(A \cup B)^c] + n(A \cap B)$$

which implies

$$n(A \cap B) = n(U) - n[A \cap B^c] - n[B \cap A^c] - n[(A \cup B)^c]$$

$$= 200 - 120 - 10 - 40 = 30$$