MA/CS 321 : Intro to Numerical Methods

Problem Set 4 Pledged

Due: Wednesday October 16, 2013

Turn in all code that you have written and turn in all output generated by your functions/scripts. Your functions/scripts must be commented and output must be formatted nicely. Turn in all code that you have written and turn in all output generated by your functions/scripts. Your functions/scripts must be commented and output must be formatted nicely.

This Assignment is Pledged. You may use your notes and books, but you are not allowed to discuss these problems with anyone but your instructor. Indicate that is your own individual effort by writing out and signing the following pledge on the first page of the assignment: “On my Honor, I have neither given nor received any unauthorized aid on this assignment.”

1. (Based on Problem 1.1.E17) Consider the following approximation

\[ \sqrt{2} \approx 1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3}. \]

This is like a floating point approximation to \( \sqrt{2} \) with a base of 60. What is the absolute error? What is the relative error? What should the next term in the sequence be?

2. (Problem 1.2.E21) Show how \( p(x) = 6(x + 3) + 9(x + 3)^5 - 5(x + 3)^8 - (x + 3)^{11} \) can be efficiently evaluated.

3. Determine the double-precision machine representation of the following decimal numbers: -9876.54321, 1/5. Can either of these numbers be represented exactly with a floating point number?

4. (1.4.C9) The computation \( 5 - \sqrt{25 + x^2} \) suffers from cancellation for \( x \approx 0 \).

   (a) Demonstrate this by writing a program that performs the computation in two ways. In the first, evaluate the computations as is, and in the second, rearrange the computation to avoid cancellation.

   (b) Generate a table with columns for \( x, y_1, y_2 \) and \( \text{abs}(y_2 - y_1)/\text{abs}(y_2) \), for \( x = 10^{-1}, 10^{-2}, \ldots, 10^{-16} \), where \( y_1 \) and \( y_2 \) corresponds to the two different ways you are doing the computation. Note that, assuming that the second computed value is closer to the truth, the last column in your table is the relative error in computing without avoiding cancellation.

   (c) Explain why the relative error becomes large as once \( x \) falls below a certain threshold.

5. (2.2.C8) Using each value of \( n \) from 2 to 9, solve the \( n \times n \) system \( Ax = b \), where \( A \) and \( b \) are define by

\[ a_{ij} = (i + j - 1)^7, \quad b_i = p(n + i - 1) - p(i - 1) \]
where
\[ p(x) = \frac{x^2}{24}(2 + x^2(-7 + n^2(14 + n(12 + 3n)))) \]

Explain what happens.

6. (4.3.E1) Using Taylor series, derive the approximation formula

\[ f'(x) \approx \frac{1}{2h}[4f(x + h) - 3f(x) - f(x + 2h)]. \]

Keep at least two error terms and show that the leading error term is of the form \( \frac{1}{3}h^2 f''(\xi) \).

If Richardson extrapolation were used to improve this approximation would the order of the error between columns increase by two or one?

7. (Based on 4.3.C8) For the following data determine the coefficients in the monomial basis and the Newton basis for the polynomial that passes through the following data:

<table>
<thead>
<tr>
<th>x</th>
<th>0.00</th>
<th>0.60</th>
<th>1.50</th>
<th>1.70</th>
<th>1.90</th>
<th>2.10</th>
<th>2.60</th>
<th>2.80</th>
<th>3.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-0.8</td>
<td>-0.34</td>
<td>0.59</td>
<td>0.59</td>
<td>0.23</td>
<td>0.1</td>
<td>0.28</td>
<td>1.03</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Plot the data and plot the interpolating polynomial. Does the resulting curve look like a good fit? Look at the First interpolation Error Theorem. Add to your plot a plot of the last term in the error theorem, \( \prod_{i=0}^{n}(x - x_i) \). Is the fit consistent with what the error Theorem would indicate. How could one avoid problems like this?