**Examples: Simple and Compound Interest**

Example 1: Suppose you make an initial deposit of $1000 into a savings account at a bank which offers a 3% yearly simple interest rate. If you make no withdrawals or deposits in the next 10 years, how much is the account worth?

Use the simple interest rate formula

\[ A = P(1 + rt) = 1000 \cdot (1 + 0.03 \cdot 10) = 1000 \cdot (1.30) = 1300 \]

Example 2: In the previous example, the bank never actually put the interest you earned on your money into your account. Rather when you go to close out your account the bank will pay back your principle ($1000) and then give you the interest you are owed for the time period ($300). Generally, this is not how banks pay interest. Instead, most banks periodically deposit your earned interest back into your account. This is known as *compounding* and this type of interest payment is called *compound interest*. Under a compound interest scheme, each time the bank deposits an interest payment into your account the base on which your interest is assessed will grow. So you’ll earn interest on both the principle and the previous deposits of interest that your bank has made.

For example, suppose that the bank in the previous example pays the same 4% interest but compounds yearly. Then after the end of the year 1, the bank will make its first interest payment and we’ll have:

\[ A_1 = 1000 \cdot (1.03) = 1030 \]

in our account. Then over the course of year 2, we are going to earn interest on $1030 not $1000. In other words, we treat the $1030 as the principle for the next year.

At the end of year 2 the bank gives us our next interest payment and the amount of money in the account is

\[ A_2 = 1030 \cdot (1.03) = 1060.90 \]

So our interest for the next year will be assessed on this $1060.90 and at the end of year 3 we’ll have:

\[ A_3 = 1060.90 \cdot (1.03) = 1092.73 \]

Continuing in this way, we find that at the end of 10 years we’ll have:

\[ A_{10} = 1343.92 \]

which is almost $44 more interest than what we earned under the simple interest rate scheme.
Example 3: Perhaps you can see the pattern in the previous example? The amount in our account after \( n \) years is given by

\[
A_n = P_{\text{initial}} \cdot (1 + r)^n
\]

But actually, this example is pretty unrealistic too. Usually banks will compound interest over a much shorter period of time, maybe quarterly, monthly, or even daily. The general formula is

\[
A = P(1 + \frac{r}{m})^{mt}
\]

where \( P \) is the principle (initial investment), \( r \) is the yearly interest rate, \( m \) is the number of conversions (number of times the bank compounds per year), and \( t \) is the number of years you keep the money in the account.

So, if instead the bank from above compounds our interest monthly, after 10 years we’ll have

\[
A = 1000 \cdot (1 + \frac{.03}{12})^{12\cdot10} = 1349.35
\]

Here monthly compounding earns us about $5 more in interest than yearly compounding.

In general, for a given interest rate more conversions (i.e. more frequent compounding) means you’ll earn more on an investment...and pay more on a loan.

Example 4: (Comparing interest rates) Suppose Bank A offers savings accounts with a 4.9 % simple interest rate, Bank B offers savings accounts with a 4.8 % rate compounded monthly. Where should you put your money?

To answer this question, we’ll consider what’s known as the \textit{effective annual yield}. The effective annual yield is the simple interest rate that gives the same yearly return as a compound interest rate.

To find this rate use the formula:

\[
\text{\( r_{\text{eff}} = (1 + \frac{r}{m})^m \)}
\]

In this example, we’d like to know what simple interest rate is equivalent to the monthly compounded rate offered by Bank B.

\[
r_{\text{eff}} = (1 + \frac{4.8}{12})^{12} - 1 = 0.049070208
\]
So the effective rate for bank B is higher than the simple rate at bank A. I’d put my money in bank B if I were you.

Note: In finance, the term “effective annual yield” is not standard. It’s probably more common to say “annual percentage yield” which is usually shortened to APY.

Example 5: (Present and Future Value) Suppose now that you are consider purchasing a security which pays a yearly 10 % fixed rate of return with daily compounding. How much should you invest now if you want to have $1,000,000 in 30 years?

To answer this question we need to consider present value and future value. In this context, the present value of our security is the initial investment $P$ and our future value is $1,000,000 which is our accumulated amount $A$. We solve the equation

$$A = P(1 + \frac{r}{m})^{mt}$$

for $P$ and get

$$P = A(1 + \frac{r}{m})^{-mt}.$$  

Substituting the given information we have:

$$P = 100000000 \cdot (1 + \frac{.10}{365})^{-365\cdot30} = 4980731.27$$

So an initial investment of $500,000 will grow to $1,000,000 in 30 years.