MathExcel Worksheet # 3: Even More Review

Reminders: Monday we will meet in the MathHouse (Columbia Avenue).
Homework A1 is due on Friday; Homework A2 is due on Monday
Worksheet 1 is due next Wednesday.

1. Carefully show that the function \( f(x) = x^3 - x \) is odd. Your answer should take the form of a chain of equalities, starting with \( f(-x) \) and ending with \(-f(x)\).

2. Recall that a function is one-to-one if it never takes on the same value twice. Mathematically, this means that if \( x \neq y \) then \( f(x) \neq f(y) \). Show that the following functions are not one-to-one by giving an example.
   (a) \( f(x) = \frac{\pi^2}{51} \)
   (b) \( f(x) = x^2 - 1 \)
   (c) \( g(x) = \frac{1}{x^2} \)
   (d) \( h(x) = 1 - 2|x| \)

3. True or false? Every function has an inverse. Explain.
4. Find the inverse for the following functions:

(a) \( f(x) = 10x - 1 \)

(b) \( f(x) = \sqrt{3x - 7} \)

(c) \( f(x) = x^3 - 7 \)

5. Let \( f(x) = \sqrt{2x + 1} \). Find \( f^{-1} \) and compute \( f \circ f^{-1}(x) \) and \( f^{-1} \circ f(x) \).

6. (Adapted from problem 20 in the book) In the theory of relativity the mass of a particle with speed \( v \) is given by

\[
f(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}}
\]

where \( m_0 \) is the mass of the particle when velocity is zero and \( c \) is the speed of light in a vacuum. It’s possible to show that this function is one-to-one
(a) Find $f^{-1}$.

(b) The function $f$ describes how mass depends on velocity. In particular, as velocity gets very close to the speed of light the mass of our particle gets very large. What relationship does $f^{-1}$ describe?

(c) Based on the form of $f^{-1}$ what can you say about the speed of the particle when its rest mass is pretty close to its traveling mass?

7. Find the exact value of each of the following. No calculator.

(a) $\log_{10}(\log_{10}(\log_{10}(10^{10^{10}})))$.

(b) $\ln\left(\frac{1}{e}\right)$

8. Most computers and calculators only have methods for evaluating natural logarithms. To find logs in different bases computers typically employ the change of basis formula: $\log_a(b) = \frac{\log_C(b)}{\log_C(a)}$. Use the change of base formula to compute the following, accurate to 3 decimal places.
(a) \( \log_2 8.4 \)

(b) \( \log_9 15 \)

9. Solve the following equations for \( x \)

(a) \( e^{2x+1} - 7 = 0 \)

(b) \( \ln(x) + \ln(x - 1) = 1 \)

(c) \( e^{ax} = Ce^{bx}, \ a \neq b. \)
10. (This is problem is adapted from problem 58 on page 72) When a camera flash goes off, the batteries immediately begin to recharge the flash’s capacitor, which stores electric charge. The amount of charge in the capacitor at time $t$ is given by

$$Q(t) = Q_0(1 - e^{-t/a})$$

where $Q_0$ is the maximum charge capacity and $a$ is a constant determined by the circuitry of the camera.

(a) What happens to the charge on the capacitor as the value of $t$ gets very large?

(b) $Q(t)$ is a one-to-one function. Find its inverse and explain its meaning.

(c) Let $a = 2$. How long does it take to recharge the capacitor to 90 % of capacity? How long does it take to recharge to 99.99 % of capacity?