MathExcel Worksheet # 4: Secants, Tangents, and Limits

**Reminders:** Homework A2 is due tonight by midnight
Worksheet 1 is due Wednesday.

1. The point $P(3,1)$ lies on the curve $y = \sqrt{x-2}$.

   (a) If $Q$ is the point $(x, \sqrt{x-2})$, find a formula for the slope of the secant line $PQ$.

   (b) Using your formula from part (a) and a calculator, find the slope of the secant line $PQ$ for the following values of $x$. Keep 4 decimal places of accuracy and be careful with rounding.
   
   i. 2.9
   ii. 2.99
   iii. 2.999
   iv. 3.1
   v. 3.01
   vi. 3.001

   (c) Using the results of part (b), guess the value of the slope of the tangent line to the curve at $P(3,1)$.

   (d) Using the slope from part (c), find the equation of the tangent line to the curve at $P(3,1)$.

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1TI-8X tip: Hit the “y=” button and put your formula from part a.) in, say, the $y_1$ position. Then go to the home screen, access the y-vars menu, and use it to type $y_1(x)$ to find the value of $y_1$ at the point $x$. You could also use the table feature.
(e) Give a reason for why these secant lines should approach a tangent line. Will secant lines always approach a tangent line? What must the function look like for this to happen?

2. (Adapted from problem 5 in the book) If a ball is thrown in the air with a velocity of 40 ft/s, its height in feet $t$ seconds later is given by $f(t) = 40t - 16t^2$.

(a) Find a general formula for the average velocity of the ball for the time period beginning at $t$ and lasting $h$ seconds.

(b) Using a calculator, find the average velocity of the ball for the time period beginning when $t = 2$ and lasting
   
   i. 0.5 second
   ii. 0.1 second
   iii. 0.05 second
   iv. 0.01 second

(c) Estimate the instantaneous velocity when $t = 2$.

(d) Can you guess the general formula for the instantaneous velocity of the ball at time $t$?
(e) On the same graph sketch the curve, any two secant lines approximating the tangent at \( t = 2 \), and the tangent line at \( t = 2 \).

3. (Problem 9 in the book) The point \( P(1, 0) \) lies on the curve \( y = \sin \frac{10\pi}{x} \).

(a) If \( Q \) is the point \((x, \sin \frac{10\pi}{x})\), use the table feature on the calculator to find the slope of the secant line \( PQ \) (correct to four decimal places) for \( x = 1.5, 1.4, 1.3, 1.2, 1.1, .9, .8, .7, .6, \) and .5. Do the slopes appear to be approaching a limit?

(b) Use a graph of the curve to explain why the slopes of the secant lines in part (a) are not close to the slope of the tangent line at \( P \).

(c) By choosing appropriate secant lines, estimate the slope of the tangent line at \( P \).

4. The point \( P(3, 0) \) lies on the curve \( f(x) = |x - 3| \). Let \( Q \) be the point \((x, |x - 3|)\).
(a) For \( x > 3 \), what is the secant line \( PQ \)?

(b) For \( x < 3 \), what is the secant line \( PQ \)?

(c) Examine the graph of \( f \). What must be true about the slope of a line that is tangent to \( f \) at \( P \)? How many such lines are there? Why might this be a problem?

5. Let \( f(x) = \begin{cases} 
  x^2 & \text{if } x \leq 0 \\
  x - 1 & \text{if } 0 < x \text{ and } x \neq 2 \\
  -3 & \text{if } x = 2 
\end{cases} \)

(a) Sketch the graph of \( f \).

(b) Compute the following.
i. \( \lim_{x \to 0^-} f(x) \)
ii. \( \lim_{x \to 0^+} f(x) \)
iii. \( \lim_{x \to 0} f(x) \)
v. \( \lim_{x \to 2^-} f(x) \)
vi. \( \lim_{x \to 2^+} f(x) \)
vii. \( \lim_{x \to 2} f(x) \)
viii. \( f(2) \)