MathExcel Worksheet # 6: Laws of Limits

Reminders: Worksheet 2 is due next Wednesday.
Homework A3 and A4 are due Monday

1. Given \( \lim_{x \to 2} f(x) = 5 \) and \( \lim_{x \to 2} g(x) = 2 \), use limit laws (justify your work) to compute the following limits. Note when working through a limit problem that your answers should be a long chain of equalities. Make sure to keep the \( \lim_{x \to a} \) operator until the very last step.

(a) \( \lim_{x \to 2} 2f(x) - g(x) \)

(b) \( \lim_{x \to 2} \frac{f(x)g(x)}{x} \)

(c) \( \lim_{x \to 2} \pi \)

(d) \( \lim_{x \to 2} [f(x)]^{\frac{3}{2}} \)

2. Calculate the following limits if they exist or determine the divergence behavior, i.e. \( \infty, -\infty \) or chaotic movement.

(a) \( \lim_{x \to -2} |x| \)

(b) \( \lim_{x \to -\pi} |x| \)
(c) \( \lim_{x \to 1} \frac{x^2 - 1}{x - 1} \)

(d) \( \lim_{x \to 2^+} \frac{x^2 - 1}{x - 2} \)

(e) \( \lim_{x \to 2} \frac{x^3 - 8}{x - 2} \) (Hint: factor.)

(f) \( \lim_{x \to 9} \frac{x - 9}{\sqrt{x} - 3} \)

(g) \( \lim_{x \to 0} \sin \left( \frac{1}{x} \right) \)

3. Show that \( \lim_{h \to 0} \frac{|h|}{h} \) does not exist by examining one-sided limits.

4. If \( \lim_{t \to 6} h(t) \) exists and \( \lim_{t \to 6} t \cdot h(t) = a \), find \( \lim_{t \to 6} h(t) \).
5. Find the value of $c$ such that \( \lim_{x \to 2} \frac{x^2 + 3x + c}{x - 2} \) exists. What is the limit?

6. Use the squeeze theorem to find (and prove) the following limits:
   
   (a) \( \lim_{x \to 0} x^2 \sin \left( \frac{1}{x} \right) \)
   
   (b) \( \lim_{x \to 0^+} \sqrt{x} e^{\sin \frac{x}{x}} \)

   (c) Given that \( 2x \leq g(x) \leq x^4 - x^2 + 2 \) for all \( x \), find \( \lim_{x \to 1} g(x) \).

7. Let \( f(x) = \frac{1}{x-5} \)
(a) Write the formula for the average rate of change of \(f(x)\) on \([1,1+h]\).

(b) Now, let your expression in a.) approach 0. This is the definition of the derivative at \(x = 1\) for \(f(x)\). What does this quantity describe?

8. Carson Palmer throws a football down the field and its position is modeled by \(p(t) = -10(t - 5)^2 + 250\). Use your answer from 7b.) and some physical intuition to find the acceleration of the football at \(t = 1\).

9. In the theory of relativity, the Lorentz contraction formula
   \[ L = L_0 \sqrt{1 - v^2/c^2} \]
   expresses the length \(L\) of an object as a function of its velocity. Find \(\lim_{v \to c^-} L\) and interpret the result. Why is the left hand limit (physically) necessary?

10. Holes vs behavior near a hole.
    (a) What’s wrong with the following equation?
        \[ \frac{x^2 + x - 6}{x - 2} = x + 3 \]
(b) Why is

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2} = \lim_{x \to 2} (x + 3)$$

ok?