MathExcel Worksheet # 7: Continuity

Reminders: Worksheet 2 is NOW due on FRIDAY.
Homework A3 and A4 are due tonight by Midnight
Exam I: Tuesday, Sept. 23, 7:30 pm -9:30 pm, CB 246.

1. Give the formal definition for a function \( f(x) \) to be continuous at a point \( x = a \).
   Give the formal definition for a function to be continuous on a domain. Then give an
   intuitive description of continuity.

2. Sketch the graph of a function that has a jump discontinuity at \( x = 2 \), is continuous
   from the right at \( x = 2 \), and has a removable discontinuity at \( x = 5 \).

3. Using the definition of continuity and the properties of limits, show that the following
   functions are continuous at the given number \( a \).
   (a) \( f(x) = \pi \), \( a = 1 \)
   (b) \( f(x) = \frac{x^2 + 3x + 1}{x + 3} \), \( a = -1 \)
(c) \( f(x) = \sqrt{x^2 - 9}, \ a = 4 \)

4. Specify the domain on which \( f \) is continuous. Use interval notation.

(a) \( f(x) = \frac{x + 1}{x^2 + 4x + 3} \)

(b) \( f(x) = \frac{x}{x^2 + 1} \)

(c) \( f(x) = \sqrt{2x - 3} + x^2 \)

(d) \( f(x) = \begin{cases} 
  x^2 + 1 & \text{if } x \leq 0 \\
  x + 1 & \text{if } 0 < x < 2 \\
  -(x - 2)^2 & \text{if } x \geq 2 
\end{cases} \)
5. For what value of the constant $c$ is the function $f$ continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} 
  cx^2 + 2x & \text{if } x < 2 \\
  x^3 - cx & \text{if } x \geq 2 
\end{cases}$$

6. Let $c$ be a number and consider the function

$$f(x) = \begin{cases} 
  cx^2 - 5 & \text{if } x < 1 \\
  10 & \text{if } x = 1 \\
  \frac{1}{x} - 2c & \text{if } x > 1 
\end{cases}$$

(a) Find all numbers $c$ such that $\lim_{x \to 1} f(x)$ exists.

(b) Is there a number $c$ such that $f(x)$ is continuous at $x = 1$? Justify your answer.

7. State the Intermediate Value Theorem.
8. Use the Intermediate Value Theorem to show that the given equation has a real solution in the specified interval.

(a) \(2x^3 + x - 5 = 0, \ [0, 2]\)

(b) \(\cos x = x, \ [0, \pi/2]\)

9. (Problem 61 in the book) Is there a number that is exactly 1 more than its cube?

10. (Problem 40 in the book) The gravitational force exerted by the earth on a unit mass at a distance \(r\) from the center of the planet is

\[
F(r) = \begin{cases} 
\frac{GMr}{R^3} & \text{if } r < R \\
\frac{GM}{r^2} & \text{if } r \geq R 
\end{cases}
\]

where \(M\) is the mass of the earth, \(R\) is its radius, and \(G\) is the gravitational constant. Is \(F\) a continuous function of \(r\)?
11. Let $f$ and $g$ be functions which are discontinuous at $a$. Give examples to show the following.

(a) $f + g$ can be discontinuous at $a$

(b) $f + g$ can be continuous at $a$

(c) $fg$ can be discontinuous at $a$

(d) $fg$ can be continuous at $a$