MathExcel Worksheet # 9: Derivatives

Reminders: Worksheet 2 is NOW due on FRIDAY.
A5 is due midnight Friday
Exam I: Tuesday, Sept. 23, 7:30 pm -9:30 pm, CB 246.
Exam I: Review Session: Funkhouser Building room 200 at 7:30 PM on Monday.

1. Let

\[ f(x) = \begin{cases} 
-x & \text{if } x \leq 0 \\
 x^2 & \text{if } x > 0 
\end{cases} \]

Show \( f \) is continuous at \( x = 0 \), but not differentiable.

2. A fixed point of a function \( f \) is a point \( x \) so that \( f(x) = x \). Use the intermediate value theorem to show that any continuous function with domain \([0, 1]\) and range \([0, 1]\) must have a fixed point.
3. Let \( s(t) = t^2 + 10t + 1 \) be the position function for a particle traveling along the x-axis. Use a definition of the derivative to find the velocity and acceleration functions for the particle.

4. Use the fact that \( \sin'(x) = \cos(x) \) to find the equation for the tangent to the curve \( \sin(x) \) at \( x = \pi \).

5. The tangent line to the graph of \( g(x) \) at \( x = 1 \) is given by \( y = 5x + 1 \). Find \( g(1) \) and \( g'(1) \).

6. Use the definition of derivative to show that if \( f(x) \) is differentiable everywhere and \( g(x) = cf(x) \), then \( g'(x) = cf'(x) \), where \( c \) is any constant.
7. Let $f$ be defined everywhere with $f(2) = 2$ and $f'(2) = 3$. Using the previous problem, find the equation of the tangent line to $g(x) = 3f(x)$ at $x = 2$.

8. Let $g(x) = |x|$. Write down the definition of derivative for $g(x)$ at $x = a$. Earlier in the semester, you showed that $\lim_{h \to 0} \frac{|h|}{h}$ did not exist by looking at left and right hand limits. What does this imply about the differentiability of $g(x)$ at $x = 0$? Graphically, why does this make sense? Use these facts to say where $h(x) = |x - c|$ will not be differentiable.

9. Let

$$ h(t) = \begin{cases} 
  at + b & \text{if } t \leq 0 \\
  t^3 + 1 & \text{if } t > 0 
\end{cases} $$

Find $a$ and $b$ so that $h$ is differentiable at $t = 0$. 
10. Let $f(t) = \sqrt{t} - 9$. Use the definition to find $f'(t)$. What is the domain and range of $f$ and $f'$. (They are NOT the same.)