MathExcel Worksheet # 3: Volumes

Reminders: 6.2 due Friday

1. Find the volume of the solid by rotating the region $R$ about the given line.

   (a) $R$ is region bounded by $y = 1 - x^2$ and $y = 0$; about $x$-axis.

   (b) $R$ is region bounded by $y = \frac{1}{x}$, $x = 1$, $x = 2$, and $y = 0$; about $x$-axis.

   (c) $R$ is region bounded by $x = 2\sqrt{y}$, $x = 0$, and $y = 9$; about $y$-axis.
(d) \( R \) is region bounded by \( y = 1 - x^2 \) and \( y = 0 \); about \( y = -1 \). (Compare to part (a), that is, which area is greater and why?)

(e) \( R \) is region bounded by \( y = e^{-x}, y = 1, \) and \( x = 2 \); about \( y = 2 \).

(f) \( R \) is region bounded by \( y = x \) and \( y = \sqrt{x} \); about \( x = 2 \).

2. Use the disc method to show that the volume of a right-circular cylinder with radius \( R \) and height \( H \) is \( \pi R^2 H \).
3. (Stewart pg. 432) Find the volume of a cap of height $H$ from a sphere with radius $R$.

4. Set up the integral expressing the volume of the solid with base being the circle $x^2+y^2 = 1$ and cross sections being perpendicular rectangles with height 3. What is this solid? Conclude that your integral expression is equal to $3\pi$.

5. (Stewart pg. 432) Find the volume of the solid with base being the ellipse $9x^2+4y^2 = 36$ and cross sections being perpendicular isosceles right triangles with hypotenuse in the base.
6. (Stewart pg. 433) Suppose a region $R$ has area $A$ and lies above the $x$-axis. (For simplicity, assume $A$ is between two continuous curves from $x = a$ and $x = b$). When $R$ is rotated about the $x$-axis, it sweeps out a solid with volume $V_1$. When $R$ is rotated about the line $y = -k$ ($k > 0$), it sweeps out a solid with volume $V_2$. Express $V_2$ in terms of $V_1$, $k$, and $A$. Note that this proves that rotating a region about a line that is farther away than another gives more volume.