MathExcel Worksheet # 5: Volumes II

**Homework Due Dates:** To clarify the due dates for the homework: When you cover a section in lecture the homework for that lecture is due on the next lecture day. So since you covered 7.1 on Monday, the homework for 7.1 is due on Weds. You’ll cover 7.2 on Weds and so the homework for 7.2 is due on Friday.

1. Let $f$ and $g$ be differentiable.
   
   (a) Differentiate $f(x)g(x)$.

   (b) Solve for $f(x)g’(x)$ and integrate both sides.

   (c) Now let $u = f(x)$ and $v = g(x)$. Find $du$ and $dv$ and use these substitutions in your formula to get the integration by parts rule.

2. Evaluate or find following using your rule from number 1 or any other appropriate method.
   
   (a) $\int \ln(x) \, dx$

   (b) $\int e^t \cos(t) \, dt$
(c) \( \int \sin^{-1}(x) \, dx \)

(d) \( \int_{1}^{2} y^4 \ln(y) \, dy \)

(e) \( \int \sin(\theta) \cos(\theta) \, d\theta \)

(f) \( \int 3^x x \, dx \)

(g) \( \int \sec(x) \tan(x) \, dx \)

(h) \( \int (\ln(x))^2 \, dx \)
(i) \( \int_{2}^{5} (x^2 - 4)e^x \, dx \)

(j) \( \int \cos(\ln(x)) \, dx \) (Hint: Make a \( u \)-substitution first.)

(k) \( \int t^n e^t \, dt \) \( n \) is any positive integer

3. Find the volume of the solid generated by rotating the region \( R \) about the given line.
   
   (a) \( R \) is region bounded by \( y = \sin(x), \ y = 0, \ x = 0, \) and \( x = \pi, \) about \( y = 0. \)
(b) \( R \) is region bounded by \( y = e^{-x}, \ y = 0, \ x = 0, \) and \( x = -1, \) about \( x = 1. \)

4. (Stewart pg. 459) If \( f(0) = g(0) = 0 \) and \( f'' \) and \( g'' \) are both continuous, show that
\[
\int_{0}^{a} f(x)g''(x) \, dx = f(a)g'(a) - f'(a)g(a) + \int_{0}^{a} f''(x)g(x) \, dx
\]

5. (Stewart pg. 459) A particle moves along a straight line with velocity \( v(t) = t^2e^{-t} \) meters per second after \( t \) seconds. How far will the particle travel during the first \( t \) seconds?