Sets and Operations on Sets

What is a set?

- A set is a collection of objects.
- An object in a set is called an element.
- Examples: Colors in the rainbow, people in the class, animals in the zoo. The numbers 1, 2, 3, ... The letters in the alphabet.
- When we talk about sets we must define the universe. The universe is all the objects that are allowed to be considered. For example, if $P$ is the set of people in the class, the universe is the set of all possible people in the world.
- Well defined: It only makes sense to talk about a set when both (a) there is universe for the set and (b) every object in the universe is either in the set or not in the set. Example: Is the collection of “Best Songs of the 1980’s” a set? How can you change the definition to make it a set?

How to describe a set.

1. Verbal description: "The set of people on student government."
2. Listing in braces: \{monkey, giraffe, kangaroo, meerkat\}.
3. Set builder notation: \{x \mid x \text{ is a letter in the alphabet}\}.

Note on listing sets: The order in which we list the elements in a set does not matter. So the set \{x, y, z\} is the same as the set \{z, y, x\}. Also, we only list each element of a set once so the sets \{1, 1, 1, 2, 3\} and \{1, 2, 3\} are the same.

Venn Diagrams You can use Venn diagrams to model sets.

Some more definitions for sets.

- If we want to say $x$ is an element of a set $A$, we will typically write $x \in A$ where the symbol “$\in$” means “in”.
- A natural number or counting number is a member of the set $N = \{1, 2, 3, ...\}$.
- The complement of a set $A$ is the set of elements in the universe set $U$ that are not in the set $A$. We will write $\bar{A}$ for the complement of the set $A$. Example: What is the complement of people in the class? What is the complement of the set \{a, b, c\}? What is the complement of a universal set? Examples 2.2 on p. 82.
- The empty set is the set with no elements in it. We write it as \{} or $\emptyset$.
- The set $A$ is subset of $B$ if every element of $A$ is also an element of $B$. If $A$ is a subset of $B$ we will write $A \subset B$. Example: the set of students in the class is a subset of the people at UK which is a subset of the people currently in Kentucky.
• The union of the sets $A$ and $B$ is the set which contains all the elements of both $A$ and $B$. The union of $A$ and $B$ is a set and we write it as $A \cup B$. In set builder:

$$A \cup B = \{x| x \in A \text{ or } x \in B\}.$$ 

Example.: $\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$

• The intersection of the sets $A$ and $B$ is the set which contains all the elements which $A$ and $B$ have in common. The intersection of $A$ and $B$ is a set and we name it $A \cap B$. In set builder:

$$A \cap B = \{x| x \in A \text{ and } x \in B\}.$$ 

Example.: $\{1, 2, 3\} \cap \{3, 4, 5\} = \{3\}$

• Two sets $A$ and $B$ are called disjoint if they have no elements in common. This is equivalent to $A \cap B = \emptyset$.

• For practice on these concepts look at the elements of the set of examples $\{2,2,2,3,2,4\}$ in your text.

Activity: Set Skits

1. Class is divided into groups of 5.
2. Each person in the group gets one of 5 different colored stickers. Say: red, green, blue, orange, yellow.
3. Each group gets a card with several set operations in list notation. Example: $\{\text{red, blue}\} \cup \{\text{yellow, orange}\}$.
4. Groups discuss their operations.
5. Each group presents their operations to the class by forming the result of the operation at the front of the room. Example: if the students have $\{\text{red, blue}\} \cup \{\text{yellow, orange}\}$, they should make the formation $\{\text{red, blue, yellow, orange}\}$ at the front of the room.