Exam II Review

The topics for the exam are sections 3.1-3.9 and 7.1. Review the text, your notes, the homework, and quizzes very carefully! Here are some problems which may be similar to problems that will appear on your exam.

1. For each of the following equations (a) Find a fundamental solution set, (b) verify that the solutions in the fundamental set are linearly independent, and (c) give the general solution.
   
   (a) $2y'' - 13y' + 6y = 0$
   (b) $y'' - 4y' + 5y = 0$
   (c) $y'' - 10y' + 25 = 0$

2. Describe how each of the solutions to the previous problems behave as $t \to \infty$. Assume that $y(0) = 0$ and $y'(0) = 1$ for each of the previous problems. Make a sketch of the solutions with these initial conditions.

3. 3.1: 10, 11, 12

4. Suppose that $W(t, g(t)) = t^2 e^t$. Find $g(t)$. Also, problem 3.2: 17.

5. Suppose $y_1(t)$ is a solution to $y'' + p(t)y' + q(t)y = 0$. Prove that $y_2(t) = cy_1(t)$ is also solution for any constant $c$. Does $y_1(t), y_2(t)$ constitute a fundamental set? Explain.

6. Find the longest interval in which the solution to
   
   $(t^2 - 1)y'' + y' + y = \sec t, \quad y(0) = 1, y'(0) = -1$

   is certain to exist.

7. Suppose $p(t)$ and $q(t)$ are continuous for all $t$. Find the unique solution to 
   
   $y'' + p(t)y' + q(t)y = 0, \quad y(0) = 0, y'(0) = 0.$

8. Prove Abel’s theorem (Theorem 3.3.2)

9. Suppose that $y_1$ and $y_2$ are a fundamental set for $y'' + p(t)y' + q(t)y = 0$. Prove that $c_1y_1(t)$ and $c_2y_2(t)$ are a fundamental set.

10. 3.4: 7-10

11. 3.5: 11-13

12. Consider a general 2nd order linear ODE given by
    
    $L[y] = y'' + p(t)y' + q(t)y = g(t)$.

    Let $y_1(t)$ and $y_2(t)$ form a fundamental set for the homogeneous equation and suppose $Y(t)$ is a particular solution of the nonhomogeneous equation. Prove that $y(t) = Ay_1(t) + By_2(t) + Y(t)$ is a solution to the inhomogeneous equation.

13. Consider the 2nd order linear ODE given by
    
    $L[y] = y'' + p(t)y' + q(t)y = g(t)$.

    Show that $Y_1(t) - Y_2(t)$ is a solution to the homogeneous equation if both $Y_1(t)$ and $Y_2(t)$ are solutions to the nonhomogeneous equation.
14. You must know how to use the method of undetermined coefficients: 3.6: 1-10 and 13-18 are good practice. I will certainly ask you to use this method for some easy nonhomogeneous terms like $g(t) = 3\cos 2t$ or $g(t) = e^{-t}$.

15. Prove the variation of parameters formula (See 3.7)

16. You must know how to use the variation of parameters formula. 3.7: 1-10 and 13-17 are good practice.

17. 3.8: 11 Don’t worry about the quasi-frequency part.

18. Spring-mass: 3.8 5-10. I will give you a spring mass problem to solve. It will very similar to one of these problems.

19. Circuits: 3.8: 12,18

20. Forced Vibration: 3.9: 5,6,9

21. Compute the Laplace transforms of $f(t) = 1$ and $g(t) = e^{at}$.

22. Compute $\mathcal{L}\{\cos t\}$. 