1. a. Given a sequence \( a_k \), the \( n \)th partial sum is the sum of the first \( n \) terms: \( S_n = \sum_{k=1}^{n} a_k \).

b. A sequence is an ordered list of numbers. A series is the sum of a sequence over the index. That is, given \( a_k \), the corresponding series is \( \sum_{k=1}^{\infty} a_k \).

c. A series converges when the sequence of partial sums \( \{S_n\}_{n=1}^{\infty} = \{\frac{1}{2}a_1, \frac{1}{2}a_1 + \frac{1}{4}a_2, \frac{1}{2}a_1 + \frac{1}{4}a_2 + \frac{1}{8}a_3, \ldots\} \) is a convergent sequence.

d. The sequence \( \{S_n\} \) converges to \( 1 \). The sequence \( \{A_n\} \) converges to 0 (otherwise, the series would diverge).

e. The geometric series converges for \( |r| < 1 \) and diverges for \( |r| \geq 1 \). If \( |r| < 1 \) then

\[ S_n = \sum_{k=1}^{n} a r^{n-1} = \frac{a}{1-r} \]

f. The harmonic sequence is \( \sum_{n=1}^{\infty} \frac{1}{n} \). The harmonic series is \( \sum_{n=1}^{\infty} \frac{1}{n} \) and it diverges.
2. \( \sum_{k=1}^{8} \left( \frac{3}{4} \right)^k = \sum_{k=1}^{8} \left( \frac{3}{4} \right)^{k-1} = \frac{3/4}{1 - 3/4} \)

**Geometric Series, converges**: \( \frac{2}{3} \)

3. Diverges, geometric series w/ \( r > 1 \).

3. \( \sum_{k=3}^{\infty} \left( \frac{1}{3} \right)^{k-1} = -1 - \frac{1}{3} + \sum_{k=1}^{\infty} \left( \frac{1}{3} \right)^{k-1} \) w/ \( |r| < 1 \)

\[ = -1 - \frac{1}{3} + \frac{1}{1 - \frac{1}{3}} \]

\[ = -1 - \frac{1}{3} + \frac{3}{2} = \frac{1}{6} \]

4. \( \sum_{n=1}^{\infty} \frac{2 + 3^n}{4^n} = \sum_{n=1}^{\infty} \left( \frac{1}{4} \right)^{n-1} + \sum_{n=1}^{\infty} \left( \frac{3}{4} \right)^n \)

\[ = \frac{\frac{1}{2}}{1 - \frac{1}{4}} + \frac{\frac{3}{4}}{1 - \frac{3}{4}} \]

\[ = \frac{2}{3} + 3 = \frac{11}{3} \]

5. \( \sum_{n=2}^{\infty} \frac{2^{n-1} + 3^{n-1}}{4^{n-1}} = -2 + \sum_{n=1}^{\infty} \left( \frac{1}{2} \right)^{n-1} + \sum_{n=1}^{\infty} \left( \frac{3}{4} \right)^{n-1} \)

\[ = -2 + \frac{1}{1 - \frac{1}{2}} + \frac{1}{1 - \frac{3}{4}} \]

\[ = -2 + 2 + 4 = 4 \]
\[ 0.04 = \frac{4}{10^2} + \frac{4}{10^4} + \frac{4}{10^6} + \frac{4}{10^8} + \ldots + \frac{4}{10^{2n}} \]

\[ = \sum_{n=1}^{\infty} \frac{4}{10^{2n}} = \sum_{n=1}^{\infty} 4 \left( \frac{1}{100} \right)^n \]

\[ = \sum_{n=1}^{\infty} \frac{4}{100} \left( \frac{1}{100} \right)^{n-1} \]

\[ = \frac{\frac{4}{100}}{1 - \frac{1}{100}} = \frac{4}{99} \]

\[ \text{Proof } 0.99999\ldots = \sum_{n=1}^{\infty} \frac{9}{10^n} = \sum_{n=1}^{\infty} \frac{9}{10} \left( \frac{1}{10} \right)^{n-1} \]

\[ = \frac{\frac{9}{10}}{1 - \frac{1}{10}} = \frac{9/10}{9/10} = 1 \]

5. For any fixed \( x \) this is a geometric series \( \text{w/ } a = 2 \) and \( r = \cos(x) \). A geometric series converges when \( |r| < 1 \) and diverges otherwise. Therefore the series converges when \( |\cos(x)| < 1 \), that is when \( x \neq k\pi \) where \( k \) is an integer.
6) Note that since \( a_n > 0 \) the sequence of partial sums is increasing. Also, the sequence of partial sums is bounded by 0 and 10^15. Therefore, \( \sum_{n=1}^{\infty} a_n \) is a bounded monotonic sequence and hence converges. Then by definition the series \( \sum_{n=1}^{\infty} a_n \) is convergent.

7) a) False. One is infinite the other is finite.
   b) False. For example, if \( a_n = (-1)^n \) then \( \lim_{n \to \infty} |a_n| = 1 \) but \( \lim_{n \to \infty} a_n \) diverges.
   c) False. For example, \( \left| \sum_{n=1}^{N} (-1)^n \right| \leq 1 \)
      but the series does not converge.
   d) False. (This is only true if \( \sum a_n = 0 \))
   e) True \( \sum_{n=1}^{\infty} a_n = a_1 + a_2 + \ldots = \sum_{n=0}^{\infty} a_n(n+1) \)