MA 114 Worksheet # 3: Series

1. Conceptual Understanding:
   (a) What is a partial sum of a sequence?
   (b) Give the definition a sequence. Give the definition of a series.
   (c) Define what it means for a series to converge.
   (d) Suppose you know that ∑_{n=1}^{∞} a_n = π. Let s_n be the n-th partial sum of \{a_n\}. What can you say about the sequence \{s_n\}? What is the limit of the sequence \{a_n\}?
   (e) Give two distinct examples of series which converge and two distinct examples of series which diverge.
   (f) The series \sum_{n=1}^{∞} a_r^{n-1}

      is called a geometric series. For what values of r does this series converge? For what values of r does this series diverge? What is the the sum if the series converges?
   (g) Write down the harmonic sequence and the harmonic series. Does the harmonic sequence converge? Does the harmonic series converge?

2. Do the following series converge or diverge? For each convergent series find the sum.

   (a) \sum_{n=1}^{∞} \left( \frac{2}{5} \right)^n 
   (b) \sum_{n=1}^{∞} \left( \frac{5}{2} \right)^{n-1} 
   (c) \sum_{n=3}^{∞} \left( \frac{1}{3} \right)^{n-1} 
   (d) \sum_{n=1}^{∞} \frac{2 + 3^n}{4^n} 
   (e) \sum_{n=2}^{∞} \frac{2^{n-1} + 3^{n-1}}{4^{n-1}} 

3. Rewrite the repeated decimal 0.0\overline{1} as an infinite series. Use the series representation to write the number as a fraction.

4. Prove that .99999999... = 1

5. For what values of x does the series \sum_{k=1}^{∞} 2 \cos(x)^{k-1} converge?

6. Let \{a_n\} be a sequence of positive numbers and s_n be the n-th partial sum of this sequence. Suppose that s_n < 10^{15} for all n. Prove that \sum_{n=1}^{∞} a_n converges.

7. Identify the following statements as true or false and explain your answers.
   (a) The sequences \{a_1,a_2,...\} and \{a_1,a_2,...,a_n\} are the same.
   (b) If \lim_{n→∞} |a_n| exists then \lim_{n→∞} a_n exists.
   (c) If the sequence of partial sums of an infinite series is bounded the series converges.
   (d) \sum_{n=1}^{∞} a_n = \lim_{n→∞} a_n if the series converges.
   (e) \sum_{n=1}^{∞} a_n = \sum_{n=0}^{∞} a_{n+1} if both of the series converge.