MA 114 Worksheet # 7: Power Series

1. Conceptual Understanding:
   (a) Define the terms power series, radius of convergence, and interval of convergence.
   (b) Find a formula for the coefficients $c_i$ of the power series
       \[ \frac{1}{0!} + \frac{2}{1!}x + \frac{3}{2!}x^2 + \frac{4}{3!}x^3 + \ldots \]
   (c) Find a formula for the coefficients $c_i$ of the power series
       \[ 1 + 2x + x^2 + 2x^3 + x^4 + \ldots \]
   (d) Consider the function $f(x) = \frac{5}{1 - x}$. Find a power series which is equal to $f(x)$ for every $|x| < 1$.
   (e) (Similar to text, prob. 39, p. 728) Suppose \( \lim_{n \to \infty} n^{1/n} = c \) where $c \neq 0$. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$.

2. Find the radius and interval of convergence for the following power series.
   (a) $\sum_{n=0}^{\infty} (5x)^n$
   (b) $\sum_{n=1}^{\infty} \sqrt{n} x^n$
   (c) $\sum_{n=1}^{\infty} \frac{x^n}{n^{1/2}}$
   (d) $\sum_{n=2}^{\infty} \frac{x^n}{3^n \ln n}$
   (e) $\sum_{n=2}^{\infty} \frac{(x - 2)^n}{n^n}$
   (f) $\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{n^4}$
   (g) $\sum_{n=2}^{\infty} \frac{x^n}{n!}$
   (h) $\sum_{n=2}^{\infty} \frac{(5x)^n}{n^3}$

3. (Text, prob. 42, p. 728) Suppose that the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$ is $R$. What is the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^{2n}$?

4. Identify the following statements as true or false. Explain your answers.
   (a) The infinite series $\sum a_n$ converges when $\lim_{n \to \infty} a_n = 0$.
   (b) An infinite series converges when the limit of the sequence of partial sums converges.
   (c) The comparison test gives the sum of a series when the series is convergent.
   (d) The harmonic series converges and the alternating harmonic series diverges.
5. Let 

\[ f(r) = \frac{1}{1 - r} \]

(a) Find a power series expansion for \( f(r) \). What is the radius and interval of convergence for this series?

(b) Let \( r = -x \) in \( f(r) \) and determine the corresponding power series expansion. What is the radius and interval of convergence?

(c) Let \( r = x^2 \) in \( f(r) \) and determine the corresponding power series expansion. What is the radius and interval of convergence?

(d) Let \( r = \frac{x^2}{9} \) in the series expansion for \( f(r) \). For what values of \( x \) does this series converge?