MA 114 Worksheet # 9: Taylor and Maclaurin Series

1. Conceptual Understanding:
   (a) Suppose that \( f(x) \) has a power series representation for \( |x| < R \). What is the general formula for the Maclaurin series for \( f \)?
   (b) Suppose that \( f(x) \) has a power series representation for \( |x - a| < R \). What is the general formula for the Taylor series for \( f \) about \( a \)?
   (c) Let \( f(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4 \). Find the Maclaurin series for \( f \).
   (d) Let \( f(x) = 1 + 2x + 3x^2 + 4x^3 \). Find the Taylor series for \( f(x) \) about \( x = 1 \).
   (e) State and explain Taylor’s theorem and Taylor’s inequality.

2. In the text, the Maclaurin series for \( \cos(x) \) is found by differentiating the series for \( \sin(x) \). Derive the Maclaurin series for \( \cos(x) \) directly, that is without differentiating the formula for \( \sin(x) \). Prove that \( \cos(x) \) is equal to the sum of its Taylor expansion for all \( x \).

3. Assume that each of the following functions has a power series expansion. Find the Maclaurin series for each. Be sure to provide the domain on which the expansion is valid.
   (a) \( f(x) = \ln(1 + x) \)
   (b) \( f(x) = x^3e^{2x} \)

4. Use a known Maclaurin series to obtain the Maclaurin series for the given function. Specify the radius of convergence for the series.
   (a) \( f(x) = x^5 \sin(3x^2) \)
   (b) \( f(x) = \frac{x^2}{1 - 3x} \)
   (c) \( f(x) = e^x + e^{-x} \)

5. Assume that each of the following functions has a power series expansion about the given point \( a \). Find the Taylor series about \( a \). Specify the radius of convergence for the series.
   (a) \( f(x) = \frac{1}{x} \quad a = 2 \)
   (b) \( f(x) = \cos(x) \quad a = \pi \)

6. Evaluate the indefinite integral \( \int \frac{e^x - 1}{x} \, dx \) as an infinite series.