Quiz 2 Solution

1. Find the following integrals.

   (a) (5 points) $\int \cos^4(x)dx$

   Solution:

   $$\int \cos^4 xdx = \frac{1}{4}\int (1+\cos 2x)^2 dx$$
   $$= \frac{1}{4}\left[\int dx + 2\int \cos 2x \; dx + \int \cos^2 2x \; dx\right]$$
   $$= \frac{1}{4}\left[x + \sin 2x + \frac{1}{2}\int (1+\cos 4x)dx\right]$$
   $$= \frac{1}{4}\left[\frac{3x}{2} + \sin 2x + \frac{1}{8}\sin 4x\right] + C$$

   (b) (5 points) $\int \frac{1}{(9-x^2)^{3/2}}dx$

   Solution: Do trigonometric substitution. Let $x = 3\sin u$. Then, $dx = 3\cos u\,du$. Then, the integral becomes

   $$\int \frac{1}{(9-x^2)^{3/2}}dx = \int \frac{1}{27(1-\sin^2 u)^{3/2}}3\cos u \; du = \frac{1}{9}\int \cos^2 u = \frac{1}{9}\tan\left[\arcsin(x/3)\right]+C = \frac{x}{9\sqrt{9-x^2}}+C$$