

Quiz 3 Solution

1. (a) (5 points) Find the partial fraction decomposition of

$$\frac{x^2 - 3x + 1}{x^3 + x}$$

Solution:

$$\frac{x^2 - 3x + 1}{x^3 + x} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1} = \frac{A(x^2 + 1) + (Bx + C)x}{x^3 + x} = \frac{(A + B)x^2 + Cx + A}{x^3 + x}$$

Comparing the powers of x , we obtain $A = 1$, $B = 0$, $C = -3$.

- (b) (5 points) Use the trapezoidal rule to estimate the integral $\int_1^4 x^2 dx$ using three steps, i.e., compute T_3 . You do not need to simplify your answer.

Solution: Since $n = 3$, $\Delta x = (b - a)/n = 1$. Then,

$$\begin{aligned} \int_1^4 x^2 dx &\approx \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + f(x_3)] \\ &= \frac{1}{2} [1^2 + 2 \cdot 2^2 + 2 \cdot 3^2 + 4^2] \\ &= \frac{43}{2} = 21.5 \end{aligned}$$

where $f(x) = x^2$.