

Quiz 5 Solution

1. (5 points) Determine if the series $\sum_{n=1}^{\infty} \frac{5^n}{3^n - 2n}$ is convergent or divergent. Clearly state the tests that you are using and show all steps!

Solution: Note that $0 \leq \frac{5^n}{3^n} \leq \frac{5^n}{3^n - 2n}$ and $\sum_{n=1}^{\infty} \frac{5^n}{3^n}$ is a divergent geometric series. Then, by Comparison

Test, $\sum_{n=1}^{\infty} \frac{5^n}{3^n - 2n}$ is divergent.

2. (5 points) Check that the following series satisfies all the conditions of the Alternating Series Test, and then apply the test.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + 5}$$

What does the test tell you about the behaviour of this series? Show your work!

Solution:

1) $a_n = \frac{1}{n^2 + 5} > 0$ for all n ,

2) $a_{n+1} = \frac{1}{(n+1)^2 + 5} \leq \frac{1}{n^2 + 5} = a_n$

3) $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{1}{n^2 + 5} = 0$.