Answer all questions and show your work. Unsupported answers may receive *no credit.* You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

Name: ______________________ Section: ______________________

1. (5 points) Use the root test to determine whether or not the series $\sum_{n=1}^{\infty} \frac{5^n}{(n^2+1)^n}$ converges or diverges. Clearly show the steps and justify your conclusion.

Solution: Let $a_n = \frac{5^n}{(n^2+1)^n}$. Then

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\left|\frac{5^n}{(n^2+1)^n}\right|} = \lim_{n\to\infty} \frac{5}{n^2+1} = 0 < 1$$

Thus by the root test the series converges.

2. (5 points) Find the radius of converges of the series $\sum_{n=1}^{\infty} \frac{n!}{2^n} x^n$. Show your work!

Solution: Let $a_n = \frac{n!}{2^n} x^n$. Then by the ratio test

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)!/2^{n+1}x^{n+1}}{n!/2^n x^n}\right| = \lim_{n\to\infty} \frac{(n+1)\,|x|}{2}$$

is ∞ if $x \neq 0$ and it is zero if $x = 0$. Thus by the ratio test the series converges when $x = 0$, so the radius of convergence $R = 0$.