We study the existence and multiplicity of entire solutions for elliptic equations, driven by a non-local integro-differential operator, which main prototype is the fractional Laplacian. The model under consideration, denoted by $P_{\lambda}$, depends on a real parameter $\lambda$ and involves two superlinear nonlinearities, one of which could be critical or even supercritical. The main theorem of the paper establishes the existence of three critical values of $\lambda$ which divide the real line in different intervals, where $P_{\lambda}$ admits no solutions, at least one nontrivial non-negative entire solution and two nontrivial non-negative entire solutions (in Journal of Differential Equations, 255 (2013), 2340-2362, joint work with G. Autuori).