REMARKS ON BHASKARA'S APPROXIMATION TO THE SINE OF AN ANGLE

BY

A. A. KRISHNASWAMI AYYANGAR

Bhaskara's rational approximation to the sine of an angle quoted in Mr. Inamdar's paper can be written $\sin\frac{\pi}{n} = \frac{16(n-1)}{5n^2-4n+4}$. It is interesting to note that this is *the best rational approximation* that can be devised. Assuming that $\frac{a+bn+cn^2}{a'+b'n+c'n^2}$ reflects the following properties of $\sin\frac{\pi}{n}$, when $n \geqslant 1$ viz.

(i) $\sin\frac{\pi}{n} = \sin\frac{\pi}{m}$ where $m = \frac{n}{n-1}$,

(ii) $\sin\frac{\pi}{n} \to 0$ as $n \to \infty$,

(iii) $\sin\frac{\pi}{n} = 0, 1, ½$ when $n = 1, 2, 6$ respectively,

we have

(iv) $\frac{(n-1)(n\cdot\overline{a+b}-a)}{a'-(b'+2a')n+n^2(a'+b'+c')} \equiv \frac{a+bn+cn^2}{a'+b'n+c'n^2}$; and

(v) $c=0,\quad a+b=0,\quad a'+2b'+4c'=a+2b,\quad a'+6b'+36c'=2a+12b.$

Using (v) in (iv) we have $a'+b'=0$, so that the last two equations in (v) may be reduced to $\left.\begin{matrix}4c'-a'=b\\36c'-5a'=10b\end{matrix}\right\}$

and hence $c' = \frac{5}{16}b,\ a' = \frac{b}{4}$ and we get the approximation

$$(n-1)\bigg/\left(\frac{5}{16}n^2-\frac{1}{4}n+\frac{1}{4}\right) \text{ or } \frac{16(n-1)}{5n^2-4n+4}$$

or more elegantly $\frac{n^2-(n-2)^2}{n^2+\frac{1}{4}(n-2)^2}$,

which is Ganesa's* variant.

Inversely we can express n in terms of $\sin\frac{\pi}{n}$ in the form

$$1-\frac{2}{n} = \sqrt{\frac{1-\sin\frac{\pi}{n}}{1+\frac{1}{4}\sin\frac{\pi}{n}}}$$

when $n > 2$, a formula which enables us to calculate readily the angle with a given sine.

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* Ganesh is an Indian Astronomer of the 16th century who wrote much to simplify Astronomy and popularise it.