

# 1 Answer Key for exam4162sp07\_v1

1.  $\diamond P = 20x + 25y \quad 0 \leq x \quad 0 \leq y \quad 3/2x + 5/2y \leq 100 \quad 7/2x + 5/2y \leq 80$
2.  $\diamond P = 16$  at  $x = 1 \quad y = 2$ .
3.  $\diamond P = 9 \quad (x, y, z) \in [0, 4, 5]$
4.  $\diamond \begin{aligned} x + y + z &= 400 \\ 35x + 40y + 45z &= 16000 \\ 40x + 35y + 40z &= 15200 \end{aligned}$
5.  $\diamond \begin{array}{cccc|cccc} 1 & 2 & 4 & 4 & 1 & 2 & 4 & 4 \\ 2 & 5 & 9 & 12 & 0 & 1 & 1 & 4 \end{array} [x, y, z] = [-2z - 4, -z + 4, z]$
6.  $\diamond$  (i) 29340908922564439200 (ii) 0.1327042205 (iii) 0.8672957795
7.  $\diamond$  (i) 19 % (ii) 36 % (iii) 67.0 %
8.  $\diamond$  (i) 0.1667 (ii) 0.1667 (iii) Yes  $[0.1389, 0.1389]$

8. Two fair dice are tossed, one red and one green.

**Let A be the event** that the sum of the numbers on top is 7.

**Let B be the event** that the number on top of the red die is **not** 1. Answer the following:

- (i) What is the probability that the sum of the numbers on top is 7, i.e. what is

$P(A)$ ?

- (ii) What is the probability that the sum of the numbers on top is 7, given that the number on top of the red die is **not** 1, i.e. what is  $P(A|B)$ ?

- (iii) Are the events  $A$  and  $B$  independent? Explain your reasoning.

**Suggestion.** Be sure to keep at least four digits of accuracy.

7. An experiment consists of casting a die and observing the number on top. It is found that 55% of the time the number on top is less than or equal to 4.

It is also observed for the same die that 64% of the time the number on top is at least 3. Answer the following questions **based on these observations**.

- (i) What is the probability that the number on top is 3 or 4 ?

- (ii) What is the probability that the number on top is 1 or 2 ?

- (iii) The experimenter concludes that this die must be loaded since a fair die would show a number less than or equal to 4 on top for % of the time.

6. State University is about to select a President, a Vice President and a five member Executive Committee to serve on the Governing Board. Instead of an election, the students are randomly selected to serve.

If there are 1200 students and if 300 of them are seniors, then answer the following questions.

- (i) How many distinct Governing Boards are possible?

**Suggestion:** If the number is very large, it is better to leave it in a product form.

- (ii) What is the probability that no senior will be selected?

- (iii) What is the probability that at least one senior will be selected?

**Suggestion.** It is recommended that you display the formulas in addition to just a final decimal answer to get partial credit.

5. (i) Consider the following system of linear equations.

$$x + 2y + 4z = 4$$

$$2x + 5y + 9z = 12$$

Write down the augmented matrix for this system of equations.

Reduce the augmented matrix to REF (the row echelon form). It is essential to show the steps of row reductions and explicitly write the row operations used.

- (ii) Using above calculations, determine all the solutions to the system of equations in  $x, y, z$  given above.

4. Set this problem up, by stating the chosen variables, the equations to be solved and **the initial augmented matrix. Do not solve the problem.**

A farmer has 400 acres of land suitable for cultivation of crops A,B,C. The cost **per acre** of cultivating the crops is \$35,\$40, \$45 respectively.

The farmer has a total of \$ 16,000 available for land cultivation.

Each acre of crop A requires 40 labor-hours, each acre of crop B requires 35 labor-hours and each acre of crop C requires 40 labor-hours.

The farmer has a total of 15,200 labor-hours available. The farmer wants to decide how many acres of each crop should be planted to use all the resources. Use  $x,y,z$  to denote the number acres planted with the three crops A,B,C respectively.

The equations to be solved are:

The augmented matrix is:

3. Here is a final tableau associated with a maximal LPP.

$x$	$y$	$z$	$s$	$t$	$u$	$P$	constants
-2	0	1	0	-4	-13	0	5
-1	1	0	0	0	1	0	4
1	0	0	1	1	4	0	2
3	0	0	0	2	13	1	9

Using your knowledge of the Simplex algorithm, determine the solution to the maximal LPP.

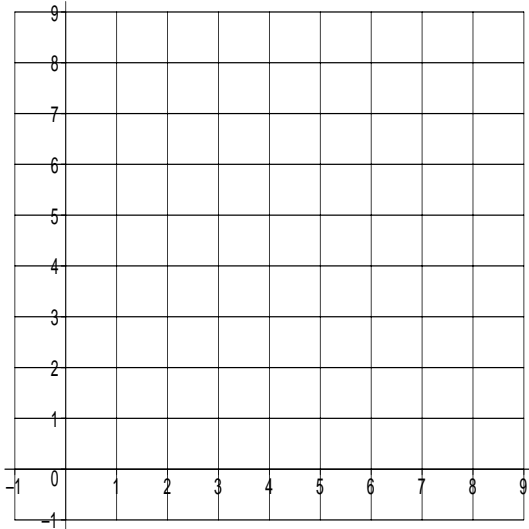
Value of P =   $(x, y, z) = ($    $,$    $,$    $)$

2. i) Sketch and shade the region described by the inequalities. Compute the coordinates of the corner points and mark them on your graph.

$$x + y \leq 3$$

$$y \geq 2$$

$$x \geq 0, y \geq 0$$



- ii) Find the maximum value of the function,  $P = 6x + 5y$  on the region.

Answer:  $P = \boxed{\phantom{000}}$  at  $x = \boxed{\phantom{000}}$ ,  $y = \boxed{\phantom{000}}$ .



1. Set this problem up, by stating the chosen variables, the function to be maximized and **all** the inequalities. **Do not solve the problem.**

Woodco makes and sells two types of wooden toys: soldiers and trains.

Each soldier sells for \$ 20, requires 1.5 hours of finishing and 3.5 hours of carpentry.

Each train sells for \$ 25, requires 2.5 hours of finishing and 2.5 hours of carpentry.

The company has 100 hours of finishing time and 80 hours of carpentry time available.

Set up a LPP whose solution will determine how many soldiers and how many trains should be made to maximize Woodco's profit.

- i) Define and explain all the variables you use.

Variables are:

- ii) Now describe the LPP explicitly.

Maximize: Profit  $P =$

Subject to:

- iii) The initial Simplex tableau is:

# DEPARTMENT OF MATHEMATICS

Ma 162 Final Exam v1 Spring 2007

**DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO.**

**Instructions:** Cell phones must be OFF and put away before you open this exam. **Be sure your name, section, and student number are filled in below. Also be sure to put your initials on each exam page.** There are 8 problems and 9 pages (including this one) on the exam.

Show your work and put your answers in the answer boxes provided. **Unsupported or misplaced answers will receive no credit and no partial credit will be given for an incorrect answer.** You may use calculators for completing numerical calculations. The test has been carefully checked and its notation is consistent with the homework problems. No additional details will be provided **during the exam.**

Problem	Maximum Score	Actual Score
1	15	
2	15	
3	7	
4	12	
5	15	
6	12	
7	12	
8	12	
Total	100	

Please fill in the information below.

NAME: \_\_\_\_\_ Section: \_\_\_\_\_

SECTION NO: \_\_\_\_\_