

DEPARTMENT OF MATHEMATICS

Ma162 Third EXAM Spring 2004
April 5, 2004

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO.

Be sure to show all work and justify your answers.

There are 5 problems and a total of 5 pages including this one. You are allowed the use of calculators.

Problem	Maximum Score	Actual Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

It is essential to fill in the following information precisely.

You may be charged points, if you cannot state your own section number.

NAME: _____ STUDENT #: _____

SECTION NO: _____

Formulas

Simple Interest: Interest. $I = Prt$, **Accumulation.** $A = P(1 + rt)$,

Compound Interest: Accumulation. $A = P(1 + i)^n$, **Present value:** $P = A(1 + i)^{-n}$

Annuity: Sum or future value $S = \frac{R((1 + i)^n - 1)}{i}$, **Present value:** $P = \frac{R(1 - (1 + i)^{-n})}{i}$

Set counts: Two set formula. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Three set formula. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$.

1. Jack has \$500 to put into savings for 3 years. He shopped around and found that Bank Uno has a savings plan offering simple interest at 5% per year. Bank Dos offers a rate of 4.75% per year, but it is compounded annually.

Help Jack make the decision by answering the following:

- (a) Find the accumulated amount Jack expects if he invests in Bank Uno.

$$A = 500(1 + (.05)(3))$$

$$A = 500(1.15)$$

$$A = 575$$

- (b) Find the accumulated amount Jack expects if he invests in Bank Dos.

$$A = 500 \left(1 + \frac{.0475}{1} \right)^3$$

$$A = 574.69$$

- (c) Explain why Jack should be happier with Bank Uno.

Uno pays \$0.31 more than Dos.

- (d) The manager of Bank Dos offers to use monthly compounding instead. Should Jack change his mind now? Justify your answer.

$$A = 500 \left(1 + \frac{.0475}{12} \right)^{3*12}$$

$$A = 576.41$$

So he should change banks, since now Dos pays \$1.41 more than Uno.

2. Three persons wish to buy a car by using different plans. Help them out by answering these questions.

- (a) Herb has already bought a car and is supposed to pay \$300 per month for the next three years. The interest rate is 6% per year compounded monthly. He wants to figure out how much the dealer would have asked for the car if he had cash. (Don't consider negotiating skills!)

$$P = 300 \left(\frac{1 - \left(1 + \frac{.06}{12} \right)^{-36}}{\frac{.06}{12}} \right)$$

$$P = 9861.30$$

- (b) Rosemary wants to buy a \$20,000 car by paying 50% cash and she wants to finance the rest by making monthly payments for a year at 5% per year compounded monthly. What would her monthly payment be?

$$10000 = R \left(\frac{1 - \left(1 + \frac{.05}{12}\right)^{-12}}{\frac{.05}{12}} \right)$$

$$10000 = 11.6812R$$

$$856.0747 = R$$

- (c) Basil wants to buy a \$20,000 car but he believes in not borrowing and is willing to wait. He wishes to save money every month in a bank account paying 5% per year compounded monthly. How much does Basil need to save per month in order to get his car in two years?

$$20000 = R \left(\frac{\left(1 + \frac{.05}{12}\right)^{24} - 1}{\frac{.05}{12}} \right)$$

$$794.09 = R$$

3. You are given the following sets. Use them to answer the questions.

$$U = \{a, b, c, d, e, f, g, h, i, j, k, l\}, A = \{a, b, d, e, f\}, B = \{b, c, d, f, h, i\}, C = \{d, e, f, g, h\}.$$

- (a) Calculate:

$$A \cap B \cap C.$$

$$\{d, f\}$$

- (b) Calculate:

$$A \cap (B \cup C) \text{ and } (A \cap B) \cup C.$$

$$\{b, d, e, f\} \text{ and } \{b, d, e, f, g, h\} \text{ respectively.}$$

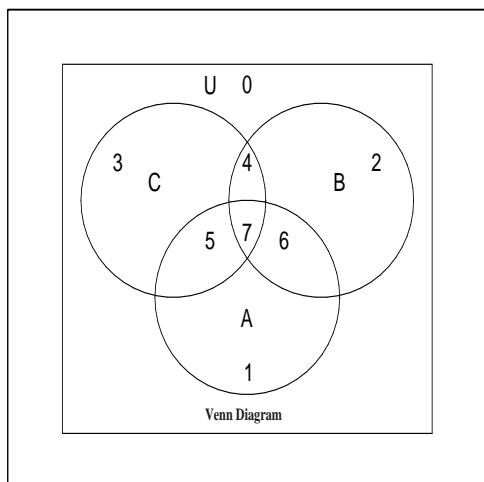
- (c) Calculate:

$$(A \cup B)^C \text{ and } A^C \cap B^C.$$

$$A \cup B = \{a, b, c, d, e, f, h, i\} \quad (A \cup B)^C = \{g, j, k, l\} = A^C \cap B^C$$

4. Consider the following Venn diagram. List the regions of the Venn diagram which represent each of the sets mentioned below. Be sure to clearly indicate which answer goes with which set.

$$A \cap B \cap C, A \cap (B \cup C) \text{ and } (A \cap B) \cup C, (A \cup B)^C \text{ and } A^C \cap B^C.$$



$A \cap B \cap C$ region = 7

$A \cap (B \cup C)$ and $(A \cap B) \cup C$ region = $\{5,6,7\}$ and $\{3,4,5,6,7\}$ respectively.

$(A \cup B)^C$ and $A^C \cap B^C$ region = $\{3,0\}$ in both cases.

5. Answer the following: ¹

- (a) In a survey of 1500 subscribers to the Lexington Herald Leader, it was found that 1250 subscribe to the daily weekday service and 800 subscribe to both the daily and the Weekend service.

How many subscribe to the weekend edition only? Let x be the number of people subscribing to weekend service. Then,

$$1500 = 1250 + x - 800$$

$$1050 = x$$

Now to find those who only use weekend service, take x and subtract every one who subscribes to both.

$$x - 800 = 1050 - 800 = 250$$

- (b) In a survey of 300 students it was found that 160 regularly went to Mathskeller, 150 used the office hours of teachers and 75 did both. How many did not get help in either place? Let x be the total number of students who got help. Then,

$$x = 160 + 150 - 75$$

$$x = 235$$

¹No actual persons were bothered in the conduction of these surveys!

So out of 300 students, 235 got help, and therefore the number of students who didn't get help is...

$$300 - 235 = 65$$

- (c) In a survey, 100 voters were asked which group they trusted to improve the economy, Democrats, Republicans or Independents. They could vote for more than one choice if desired.

Of these, 12 declared that they don't trust anyone to improve the economy and nobody voted for all three.

There were 40 votes of confidence for Democrats, 38 for Republicans and 55 for the Independents.

- Determine the number of voters who were not loyal to one group, i.e. those who cast votes for more than one group.
- Use your answer to determine the total number of voters loyal to one of the three groups.

Let x be the number of people not loyal to a single group. Then since nobody voted for all three we have,

$$x = n(D \cap R) + n(D \cap I) + n(I \cap R)$$

Then we find an equation for the total number of voters, which is...

$$100 = 12 + n(D) + n(R) + n(I) - n(D \cap R) - n(D \cap I) - n(R \cap I)$$

$$100 = 12 + n(D) + n(R) + n(I) - (n(D \cap R) + n(D \cap I) + n(R \cap I))$$

$$100 = 12 + n(D) + n(R) + n(I) - x$$

$$100 = 12 + 40 + 38 + 55 - x$$

$$45 = x$$

Now, since people either voted for nobody, 1 party, or 2 parties, we know that the total voters is equal to the sum of those parts...

$$100 = 12 + 45 + y$$

where y is the number of people voting for only one party.

$$43 = y$$