DEPARTMENT OF MATHEMATICS

Ma 162 First Exam February 4, 2008

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO.

Instructions: Be sure your name, section number, and student ID are filled in below. Cell phones must be OFF and put away before you open this exam. You may use calculators (including graphing calculators, but no laptops or cellphone calculators) for checking numerical calculations, but you must show your work to receive credit.
Put your answers in the answer boxes provided, and show your work.
If your answer is not in the box or if you have no work to support your answer, you will receive no credit.
The test has been carefully checked and its notation is consistent with the homework problems. No additional details will be provided during the exam.

Please fill in the information below.

NAME: ______________________________ **Section:** ____________
Last four digits of Student ID: __________________

---

1. The equation for Fahrenheit temperature in terms of Centigrade temperature is $F = \frac{9}{5}C + 32$.

   a) When is the Fahrenheit temperature equal to 4 times the Centigrade temperature?

   Answer: Set up the equation $F = 4C$ and use the known formula to write $\frac{9}{5}C + 32 = 4C$.

   Rearrange this as
$$(4 - \frac{9}{5})C = 32 \text{ or } C = 32\frac{5}{20-9} = \frac{160}{11}.$$

   They ask for the value of F, so plug this into the known formula again:
$$F = \frac{9}{5}\frac{160}{11} + 32 = \frac{288 + 11 \cdot 32}{11} = \frac{640}{11} = 58.181818.$$

   b) Can 5 times the Fahrenheit temperature ever be 8 more than 9 times the Centigrade temperature? ($5\,F = 9\,C + 8$) Why or why not? **Answer:**No!

   If this were true, then we have two equations $F = \frac{9}{5}C + 32$ and $5F = 9C + 8$.

   The first is equivalent to $5F = 9C + 160$ and this is inconsistent with $5F = 9C + 8$.

2. A tourist travels from city A with coordinates $(0, 0)$ to city C with coordinates $(12, 10)$. He must pass through **exactly one of the cities** $B(7, 5)$ or $D(5, 7)$ along the way. Assume he travels the straight line between cities.

   a) Which city should he pass through (B or D) in order to minimize his trip distance from A to C?

   Answer: We see
$$d(A, B) + d(B, C) = \sqrt{7^2 + 5^2} + \sqrt{(12-7)^2 + (10-5)^2} = \sqrt{74} + \sqrt{50}.$$

Also

$d(A,D)+d(D,C) = \sqrt{5^2+7^2}+\sqrt{(12-5)^2+(10-7)^2} = \sqrt{74}+\sqrt{49+9} = \sqrt{74}+\sqrt{58}.$

So clearly, the route through B is shorter and he should take that.

b) What is the total minimum length of his trip from A to C?

As calculated, the answer is $\sqrt{74}+\sqrt{50} = 15.67$.

3. Point A has coordinates $(6,1)$,and point B has coordinates $(0,8)$.

   a) What is the distance from A to B and what is the slope of the line through A and B?

   distance: $\sqrt{(0-6)^2+(8-1)^2} = \sqrt{36+49} = \sqrt{85} = 9.2195$.

   slope: $\dfrac{8-1}{0-6} = -\dfrac{7}{6}$.

   b) Find the number y so that the point C with coordinates $(9,y)$ lies in the first quadrant and triangle ABC is a right triangle with right angle at A. (Note: The coordinates of A and B were given at the top of the problem.)

   We equate the product of the slopes of AB and AC to -1.

   Thus:

$$\frac{7}{-6}\frac{y-1}{3} = -1.$$

   This simplifies to $y-1 = \frac{18}{7}$ or

$$y = 1+\frac{18}{7} = \frac{25}{7} = 3.5714.$$

4. The cost function for a manufacturer is $C = 4\,x + 6600$, where x is the number of units produced per month and C is measured in dollars. His revenue is \$11 per unit.

   a) Determine the manufacturer's profit $P = m\,x + b$, assuming he can sell all the units he manufactures.

   Answer: We have:

$$P(x) = R(x) - C(x) = 11x - (4x+6600) = 7x - 6600.$$

   b) Determine the breakeven value for x and the breakeven cost C at that value for x.

   Answer: We solve for $P(x) = 0$ to get $x = \frac{6600}{7}$.

   The corresponding cost is $C(x) = 4\frac{6600}{7} + 6600 = \frac{72600}{7}$.

5. In a free market, the supply equation for a supplier of wheat is $x = 40\,p + 100$ where p is in dollars and x is in bushels. When the price is \$1 per bushel the demand is 540 bushels. When the price goes up to \$10 per bushel the demand is 0 bushels. Find the equilibrium price and the number of bushels supplied at the equilibrium price.

   Answer: Assume a demand function $x = ap + b$ where we have naturally used the same letter x for both demand and supply.

   Use the given information to get two equations

   $$540 = a(1) + b \text{ and } 0 = a(10) + b.$$

   Subtracting, we get $540 = -9a$ or $a = -60$. The second equation now gives $0 = -60(10) + b$ or $b = 600$. Thus $x = -60p + 600$.

   Now for the equillibrium price, we solve

   $$x = 40p + 100 = -60p + 600 \text{ which gives } 100p = 500 \text{ or } p = 5.$$

   That gives $x = 40(5) + 100 = 300$ as the supply at the equillibrium.

6. For what value of k is the system

   $$\begin{cases} x - 2\,y + z = 1 \\ 2\,x + y + 3\,z = 0 \\ y + kz = 0 \end{cases}$$

   inconsistent (i.e. has no solution)?

   Answer: Make an augmented matrix and start turning it into REF.

   $$\begin{array}{c} \begin{matrix} x & y & z & RHS \end{matrix} \\ \left[\begin{array}{rrr|r} 1 & -2 & 1 & 1 \\ 2 & 1 & 3 & 0 \\ 0 & 1 & k & 0 \end{array}\right] \end{array}.$$

   Start with

   $$\begin{array}{c} \begin{matrix} x & y & z & RHS \end{matrix} \\ \left[\begin{array}{rrr|r} 1 & -2 & 1 & 1 \\ 2 & 1 & 3 & 0 \\ 0 & 1 & k & 0 \end{array}\right] \end{array} \xrightarrow{R_2 - 2R_1} \begin{array}{c} \begin{matrix} x & y & z & RHS \end{matrix} \\ \left[\begin{array}{rrr|r} 1 & -2 & 1 & 1 \\ 0 & 5 & 1 & -2 \\ 0 & 1 & k & 0 \end{array}\right] \end{array}.$$

   Next, do:

   $$\begin{array}{c} \begin{matrix} x & y & z & RHS \end{matrix} \\ \left[\begin{array}{rrr|r} 1 & -2 & 1 & 1 \\ 0 & 5 & 1 & -2 \\ 0 & 1 & k & 0 \end{array}\right] \end{array} \xrightarrow{R_3 - \frac{1}{5}R_2} \begin{array}{c} \begin{matrix} x & y & z & RHS \end{matrix} \\ \left[\begin{array}{rrr|r} 1 & -2 & 1 & 1 \\ 0 & 5 & 1 & -2 \\ 0 & 0 & k - \frac{1}{5} & \frac{2}{5} \end{array}\right] \end{array}.$$

If $k - \frac{1}{5} \neq 0$ then we have an REF with three pivots and hence a unique solution. If $k - \frac{1}{5} = 0$, then the last equation becomes inconsistent. So the answer is $k = \frac{1}{5}$.

7. Given the system of equations $\begin{cases} -x + y + 3\,z = 0 \\ 2\,x - y - 4\,z = -1 \\ 2\,x - 2\,y - 5\,z = 2 \end{cases}$

a) Write the augmented matrix for the system.

Answer:

$$\left[\begin{array}{rrr|r} x & y & z & RHS \\ -1 & 1 & 3 & 0 \\ 2 & -1 & -4 & -1 \\ 2 & -2 & -5 & 2 \end{array}\right].$$

b) Carry out standard row reductions to convert the augmented matrix to REF(row echelon form). Be sure to describe your reductions in standard notation. Just giving the final form will receive no credit.

$$\left[\begin{array}{rrr|r} x & y & z & RHS \\ -1 & 1 & 3 & 0 \\ 2 & -1 & -4 & -1 \\ 2 & -2 & -5 & 2 \end{array}\right] \xrightarrow{R_2+2R_1, R_3+2R_1} \left[\begin{array}{rrr|r} x & y & z & RHS \\ -1 & 1 & 3 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right].$$

We are done with REF since the pivot position sequence (p.p.) is now $(1, 2, 3)$.

8. You are given the system of equations $\begin{cases} -x + y - 3\,z = -3 \\ 2\,x - y + 5\,z = 5 \\ 2\,x - 2\,y + 7\,z = 8 \end{cases}$

Here is the augmented matrix of the system reduced to a row echelon form.

$$\begin{pmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

Use it to decide if the system has **no solutions, 1 solution, or more than 1 solution**. Give your reason and describe the solution completely.

The final form can be rewritten as:

$$\left(\begin{array}{rrr|r} x & y & z & RHS \\ 1 & 0 & 2 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right).$$

Thus, each x, y, z is a pivot variable and there is no pivot on RHS. Therefore the equations are consistent and have a unique solution. The solution can be found by back substitution:

From the **third** equation: $z = 2$. From the **second** equation: $y - z = -1$ or $y = z - 1 = 1$. From the **first** equation: $x + 2z = 2$ or $x = 2 - 2z = -2$.

So the **complete solution** is $(x, y, z) = (-2, 1, 2)$.