

WHS Problems C3.

Ma 162 Fall 2009

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Counting formulas.

- **Q.1.** Given that A, B and C are sets with 95, 69 and 85 members respectively,
If $B \cap A$ has 44 members, then $B \cup A$ has ??? members.
- **Answer:** Use the two set formula
$$n(B \cup A) = n(B) + n(A) - n(B \cap A) = 69 + 95 - 44 = 120.$$
- ii) If it is further known that $C \cap A$ has 57 members, then $C \cup A$ has ??? members.
- **Answer:** Similar formula gives:
$$n(C \cup A) = n(C) + n(A) - n(c \cap A) = 85 + 95 - 57 = 123.$$
- iii) If, in addition $B - C$ has 39 members, then $C \cap B$ has ??? members.
- Note that $n(B) = n(B - C) + n(C \cap B)$, so
 $69 = 39 + n(C \cap B)$. Hence the answer is 30.

Q.1. continued.

- iv) Finally, if we are given that the intersection of all three sets A , B and C has 17 members, then the union of all three sets has ??? members.
- **Answer:** Apply the three set formula and numbers known from above to get:
$$n(A \cup B \cup C) = 95 + 69 + 85 - (44 + 30 + 57) + 17 = 135.$$

Question 2.

- **Q.2.** Your favorite restaurant, Wally's Fine Dining, has a dinner menu with 6 appetizers, 11 entrees, and 8 desserts. A dinner at Wally's consists of 1 appetizer, 1 entree, and 1 dessert. What is the largest number of days could you eat dinner at Wally's without ever ordering the exact same meal?
- **Answer:** Imagine a choice card with three boxes marked appetizer, entree and dessert. They can be respectively filled with 6, 11, 8 choices and by multiplication principle, the answer is the product:
 $(6)(11)(8) = 528.$

Further Counting.

- **Q.3.** Manjula is extremely fashionable; so she can't stand wearing the same outfit twice to one job. She owns 6 shirts, 3 pairs of pants, and 9 pairs of shoes. If she works 270 days at her current job, how many more shirts must she get to have enough so that she will never have to wear the same outfit twice?
- **Answer:** Suppose she buys x new shirts. Then by multiplication principle, she would be good for $(6 + x)(3)(9) = 162 + 27x$ days.
- We want this answer to become at least 270. Clearly $x = 4$ works! $(6 + 4)(3)(9) = 270$.

Question 4.

- **Q.4.** Three friends are playing a murder mystery game. To win, they must correctly guess which suspect murdered someone, what room they committed the crime in, and what weapon was used to do it. One of the friends starts guessing combinations (without repeating any he already guessed). What is the maximum number of guesses this friend must make to win the game if there are 7 possible weapons, 4 possible suspects, and 5 possible rooms to commit the murder in?
- **Answer:** The problem is a simple multiplication principle again, so $(7)(4)(5) = 140$.

Question 5.

- **Q.5.**

In a local dog show, there are prizes for first, second, and third place. If there are 14 dogs competing, how many ways are there to award the prizes (assuming no dog can win two prizes)?

- **Answer:** A naive guess is that it is a simple multiplication problem, make three boxes marked No. 1, 2 3 and fill in names of one othe 14 dogs in them in different ways.
- However, a little thought shows that while the No. 1 box has 14 choices, the No. 2 box can be filled in only 13 ways and the No. 3 box has only 12 choices left. The answer is, therefore $(14)(13)(12) = 2184$.

Question 6.

- **Q.6.** Commercial radio station call signs (i.e. WZZQ) are assigned following certain rules. The first letter must be either "W" or "K" and the last three letters can be anything as long as they aren't all the same. How many possible radio station call signs are possible?
- **Answer:** We imagine four slots to be filled with letters. The first slot has only 2 choices W, K . We ignore the restriction for a minute and count the choices for the remaining slots as 26 each. So this tentative answer is $(2)26^3$.
- Bringing in the restriction, we realize that for each starting letter (W or K) we throw out 26 slots choices like AAA, BBB etc. Thus the number of discards is $(2)(26)$.
Final answer $(2)(26^3) - (2)(26) = 35100$.

Q. 6 continued.

- How many different radio station call signs can you have with the first letter W and the second letter different from B ?
- **Answer:** We follow the same idea. The first slot has a single choice W , the second has 25 (other than B and if we ignore the restriction, then we have 26 each for the last slots. This gives $(25)(26^2)$. We subtract off the count of the signs $WAAA, WCCC, WDDD, \dots$ etc and this number is 25. Final answer is: $25(26^2) - 25 = 16875$.

Question 7.

- **Q.7.**

Some prisoners are trying to make a jail break. To open the door to their freedom, they have to put a series of 10 switches in the one configuration that will open the door. Each switch can be either "OFF" or "ON" .(All of them are currently "OFF" and the door is locked.) It takes them 1 minute to test each new configuration. If they have 17 hours 1 minutes before they are discovered missing and the alarm sounds, are they guaranteed to open the door before the alarm sounds?

- In either case, calculate the exact number of spare minutes left when they open the door or the number of minutes they are short by: Ans:

- **Answer:** First we compute the number of choices they have to make. There are 10 switches and each has two choices, so the answer is $2^{10} = 1024$.

Questions 7,8.

- **Q.7 continued.**

But note that one of these is already noticed and is a wrong choice, so there are 1023 choices left.

They have $(17)(60) + 1 = 1021$ minutes, so they cannot be guaranteed to escape.

they fall short by 2 minutes!

- **Q.8.** License plates in a certain state all have the same format.

They all have 3 numbers followed by 2 letters.

How many license plates can the state issue before being forced to reuse numbers from older plates?

- **Answer:** This is simple multiplication principle problem: There are five slots to fill with 10 choices each for the first three slots and 26 each for the last two.

So the answer is $(10^3)(26^2) = 676000$.