

Lecture on sections 1.1,1.2

Ma 162 Fall 2009

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Basic Definitions.

- A linear function of one variable x is a function $f(x) = mx + c$ where m, c are constants. Its graph is a straight line, hence it is called linear.

Example: $f(x) = 3x + 4$.

- A linear function of two variables x, y is of the form $f(x, y) = ax + by + c$. Its graph is a plane in three space.

Example: $f(x, y) = 3x + 4y + 5$.

- A natural generalization is a linear function of n variables $f(x_1, x_2, \dots, x_n) = a_1x_1 + a_2x_2 + \dots + a_nx_n + b$ where a_1, a_2, \dots, a_n, b are constants.

Example: $f(x, y, z) = 3x + 4y - 5z + 7$.

- These functions are useful in many applications.
- What are examples of functions which are **not** linear?

Example:

$f(x) = x^2 + 3x, g(x, y) = x^3 - y^3, h(x, y, z) = xy + yz + zx$.

Real Life functions

Here are some examples of real life functions which behave like linear functions.

- **Distance traveled** If the time interval is short or if an object is moving without acceleration, then $s = at + b$ describes the distance traveled at time t . The coefficient a is the constant **velocity**. Its sign describes if the object is moving away or coming closer.
- **Revenue, Cost and Profit Function.** If x is the number of units sold or manufactured, then we have three natural functions associated with it.
- The cost function is $C(x) = cx + f$ where c is the production cost per unit and f is the fixed cost.
- The revenue function is $R(x) = px$ where p is the price per unit.
- The profit function: $P(x) = R(x) - C(x)$ or $(p - c)x - f$.

Lines or the linear functions of one variable.

We now review how to study the properties of a linear function of one variable x by known geometric properties of its graph, the line.

- **Plane coordinates** Recall that points in the plane are pairs of numbers (x, y) , these are the x and y coordinates respectively.

A point named P with coordinates $(2, 3)$ can be denoted as $P(2, 3)$.

- **Distance Formula.** Recall that the distance between two points $P(a_1, b_1)$ and $Q(a_2, b_2)$ is given by the formula

$$d(P, Q) = \sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2}.$$

Example. The distance between $P(2, 3)$, $Q(-1, 7)$ is:

$$d(P, Q) = \sqrt{(-1 - 2)^2 + (7 - 3)^2} = \sqrt{9 + 16} = 5.$$

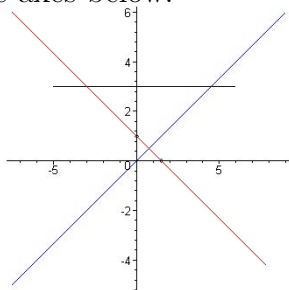
Graph of a function.

The graph of a function $y = f(x)$ consists of all points $P(x, y)$ for which $y = f(x)$.

Example. The graphs of the lines

$$y = \frac{2x}{3}, y = 1 - \frac{2x}{3}, \text{ and } y = 3$$

are shown on the same axes below.



More about lines

- As the graph of a linear function, a line has the equation $y = mx + c$.
- A vertical line does not appear as the graph of linear function. Indeed, it cannot be the graph of **any function**. A vertical line is described by an equation of the form $x = p$ where p is a constant.
- We may combine these two cases and say that the **general equation of a line** in the plane is of the form:

$$ax + by + c = 0$$

where at least one of a, b are non zero.

Recognizing a Line.

- ① Given a line $ax + by + c = 0$, if $b \neq 0$, then it is not vertical. <Indeed,> we can rewrite it as

$$by = -ax - c \text{ or } y = \frac{-a}{b}x + \frac{-c}{b}.$$

Example.

The line $2x - 3y + 5 = 0$ is rewritten as $y = \frac{-2}{-3}x + \frac{-5}{-3}$ or

$$y = \frac{2}{3}x + \frac{5}{3}.$$

- ② The slope of the general line $ax + by + c = 0$ is $m = -\frac{a}{b}$ when $b \neq 0$.

If $b = 0$ then the line is vertical and slope is infinite. We sometimes allow an equation $m = \infty$ to express this idea.

- ③ Positive slope describes a line rising to the right, negative slope describes a line falling to the right, zero slope gives a horizontal line.

Equations of Lines.

- Given two distinct points in the plane, there is a unique line joining them.

If the two points are (a_1, b_1) and (a_2, b_2) , then the slope of this line is:

$$m = \frac{b_2 - b_1}{a_2 - a_1}.$$

Example: The slope of the line joining $(2, 3)$ and $(-1, 6)$ is:

Formula for Slope $\frac{6 - 3}{-1 - 2} = -1.$

More Equations of Lines.

- The equation of a line with a given slope m and passing through a point (a, b) is:

Point Slope Formula. $y - b = m(x - a)$

Example: The slope of the line joining $(2, 3)$ and $(-1, 6)$ is:
We use the calculated slope -1 and the point $(2, 3)$ to get:

$$y - 3 = -1(x - 2) \text{ simplifies to } y = -x + 5 \text{ or } x + y - 5 = 0.$$

- As expected, all formulas need a special handling when the slope becomes infinite, i.e. when the line is vertical. The reader should make this adjustment as necessary.

Intercepts of Lines.

- The intercept of a line refers to its intersection with an axis, when defined.
- Thus, the x -intercept of a line $ax + by + c = 0$ is given by its intersection with the x -axis (i.e. $y = 0$) and is clearly equal to $-\frac{c}{a}$.

When $a = 0$, this is infinite if $c \neq 0$ and undefined when $c = 0$, i.e. when the line is the whole x -axis.

- Similarly, the y -intercept of a line $ax + by + c = 0$ is given by its intersection with the y -axis (i.e. $x = 0$) and is clearly equal to $-\frac{c}{b}$.

When $b = 0$, this is infinite if $c \neq 0$ and undefined when $c = 0$, i.e. when the line is the whole y -axis.

- Note that for the line equation $y = mx + c$, the y -intercept is c .

Intercepts form of Lines.

- **Example:** What are the x and y intercepts of $2x - 3y + 5 = 0$?

Answer: The x -intercept is $-\frac{5}{2}$ and the y -intercept is $\frac{5}{3}$.

- Sometimes, it is desirable to get the equation of a line with given x -intercept p and y -intercept q .

A nice formula is:

Intercept Form.
$$\frac{x}{p} + \frac{y}{q} = 1$$

Example: What is the equations of a line with x -intercept -3 and y -intercept 2 ? **Answer:**

$$\frac{x}{-3} + \frac{y}{2} = 1 \text{ which simplifies to } 2x - 3y + 6 = 0.$$

- As before, special cases when p or q are zero must be handled separately. These correspond to lines through the origin and it is best to use alternate formulas.