Supplementary Lecture on section 2.4

Ma 162 Fall 2010

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Definition of a Matrix.

The aim of this supplementary lecture is to help you understand the "practical meaning" of the matrix product. We discuss the related problems in homework B1. We assume that you are familiar with the usual matrix operations already. Compare this lecture with Lecture 3, where we did similar work, without bringing in the formalism of matrices. The following is based on a WHS problem in B1. Suppose that Bill holds 12, 12, 15, 16 stocks respectively of IBM, Google, Toyota and Texaco. Suppose that Jim holds 15, 11, 16, 17 stocks of same companies respectively. If the prices of these stocks are 5, 1, 2, 3 respectively, then we use matrices to organize the information and calculate.

Organization of the data.

- We begin by building a matrix to record the above data as follows. We have added in row and column titles for understanding, but they do not take part in matrix operations.
- The holding Matrix.

	IBM	Google	Toyota	Texaco
Bill	12	12	15	16
Jim	15	11	16	17

• The price matrix.

• We respectively write $A = A_{2\times 4}$ and $B = B_{4\times 1}$ for the two matrices.

Meaning of the product AB.

• Note that we have

$$A = \begin{bmatrix} 12 & 12 & 15 & 16 \\ 15 & 11 & 16 & 17 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 1 \\ 3 \\ 2 \end{bmatrix}.$$

• And we have:

$$AB = \left[\begin{array}{c} 12 \cdot 5 + 12 \cdot 1 + 15 \cdot 3 + 16 \cdot 2 \\ 15 \cdot 5 + 11 \cdot 1 + 16 \cdot 3 + 17 \cdot 2 \end{array} \right] . = \left[\begin{array}{c} 149 \\ 158 \end{array} \right] .$$

The calculation clearly shows that it is giving us the values of the holding of Bill and Jim respectively. This answers the questions.

Another Example.

- The following is based on another WHS problem in B1.
- we are given matrices (with informative headers):

Demands	Type I	Type II	Type III	Type IV	
IL	22	22	5	23	
OH	17	25	24	23	
SC	14	21	12	20	
TN	23	12	12	18	

and

Profits	Thousand \$
Type I	19
Type II	11
Type III	15
Type IV	9

Meaning of the product.

- As before, if we name the matrices $A = A_{4\times4}$ and $B = B_{4\times1}$, then AB has type 4×1 . Its four rows correspond to the four rows of A, thus belong to the indicated states.
- The resulting column gives the profit totals.
- Thus the (2,1) entry of AB gives the profit made in $\mathrm{OH}(\mathrm{IO})$ and is equal to:

$$17 \cdot 19 + 25 \cdot 11 + 24 \cdot 15 + 23 \cdot 9 = 1165.$$

- Similar interpretations can be made by using the meanings of the rows and columns of the matrix.
- Various examples in Chapter 2.4 should be reviewed to understand this concept.