

Lectures on Sets.

Ma 162 Spring 2009

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Set Notations.

- **Sets** are collections of objects. For convenience and to avoid some logical problems, we always agree to a universal set U and understand that all sets under discussion are contained in it.
- An empty set is denoted by the symbol \emptyset . A set can be described in essentially one of two ways:

- * An explicit list (roster notation) as in $A = \{1, 2, 3\}$
- * or by a defining property (set builder notation)

$$A = \{x \mid x \text{ is one of the first three natural numbers.}\}$$

- We can build new sets from given sets, say A, B by repeated application of one of these operations:

$$\text{Union } A \cup B \quad \text{Intersection } A \cap B \quad \text{Subtraction } A \setminus B.$$

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Union $A \cup B$ **Intersection** $A \cap B$ **Subtraction** $A \setminus B$.

- Let $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 5, 6\}$. We define and illustrate the various set operations.
- $A \cup B$ is the set obtained by putting elements of A as well as B together. By convention, repeated elements are written only once. So $A \cup B = \{1, 2, 3, 4, 5, 6\}$.
- It is worth noting that $X \cup U = U$ and $X \cup \emptyset = X$ for all sets X .
- We can also take the union of several sets and the final result does not depend on the order of taking the unions. In fancy words, this says that the union is an associative and commutative binary operation. Thus, if $C = \{2, 4, 8\}$, then

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Intersection and Subtraction.

- Let $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 5, 6\}$. We define and illustrate the various set operations.
- $A \cap B$ is defined as the set of common elements of A and B . Thus, for our example, it gives $A \cap B = \{2, 3\}$.
- $A \setminus B$ is defined by listing elements of A **except for** elements of B . This may also be **written as** $A - B$. Thus, for our example, $A \setminus B = \{1\}$. Note that by the same definition $B \setminus A = \{4, 5, 6\}$.
- The set $U \setminus A$ is also called the **complement** of A and has a special notation A^C . Thus, for our example

$$A^C = \{1, 2, 3, 4, 5, 6, 7, 8\} \setminus \{1, 2, 3\} = \{4, 5, 6, 7, 8\}.$$

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calculations in Set Builder notation.

- For finite sets, the lister notation is easy to work with and set operations are easily handled. For large sets or infinite sets, one has to use the set builder notation and the calculation of set operations also has to be understood in the same way. Here is an illustration.
- Let U be the set of all integers $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$.
Let

$$A = \{x \mid x = 2t \text{ for some } t \in U\}, B = \{x \mid x = 5t \text{ for some } t \in U\}.$$

- We claim the following:

$$A \cup B = \{x \mid x = 2t \text{ or } x = 5t \text{ for some } t \in U\}.$$

A little thought shows that the set consists of all even numbers as well as all elements of $\{\pm 5, \pm 15, \pm 25, \pm 35, \dots\}$.

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Continued Set Builder Notation.

- With the above definition of U, A, B we see that

$$A \cap B = \{x \mid x = 2t \text{ and } x = 5t \text{ for some } t \in U\}.$$

This set is easily seen to be all multiples of 10.

- The complement A^C is easily seen as the set of all odd integers and can be described as $\{2t + 1 \mid t \in U\}$.
- Also:

$$A \setminus B = \{x \mid x = 2t \text{ and } x \neq 5t \text{ for any } t \in U\}.$$

These are all even integers which don't end in a zero!

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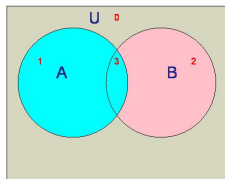
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Venn Diagrams.

- Often a graphical technique helps us understand the set operations and helps in deducing properties of calculated sets.
- If one is working with two sets A, B , then it is easy to see that any combination of operations between them results in a set which is a union of some of the following sets in a Venn diagram:

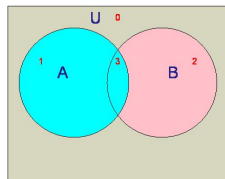


2 set Venn Diagram

- We shall find it convenient to number them so that the set 1 is $A \setminus B$, set 2 is $B \setminus A$, set 3 is $A \cap B$ and set 0 is $(A \cup B)^C$.
- This simple observation can be used to prove many facts about combinations of two sets as shown below.

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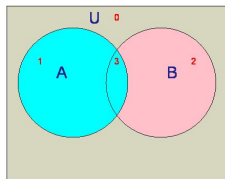


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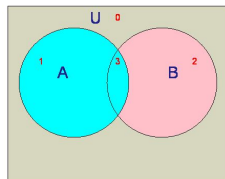


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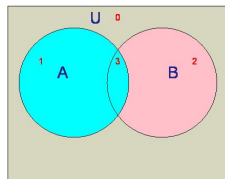


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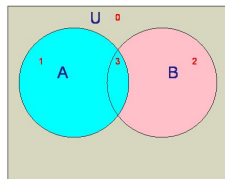


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Using the Two-set Venn Diagram.

- We can use the set numbering to “prove theorems” about two sets. Thus, we show how to prove:

$$(A \cup B)^C = A^C \cap B^C.$$

- Note from the Venn diagram that $A \cup B$ is composed of the pieces 1, 2, 3 so its complement is the piece 0. This says that the set on the left hand side is the 0 piece.
- From the Venn diagram, we also see that A^C and B^C are composed of sets “2, 0” and “1, 0” respectively. Thus, their intersection is also the piece “0”.
- The same technique can be used to prove or disprove any statements about two sets.

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- From the Venn diagram, we also see that A^C and B^C are composed of sets “2, 0” and “1, 0” respectively. Thus, there intersection is also the piece “0”.
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- For any finite set X , let $n(X)$ denote the number of elements in X . For two sets A, B we denote by d_1, d_2, d_3, d_0 the respective number of elements in the Venn diagram pieces 1, 2, 3, 0.
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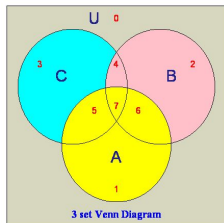
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- Here is a similar idea to handle three sets A, B, C .



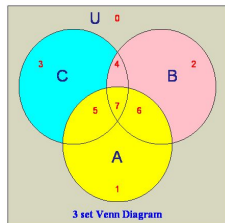
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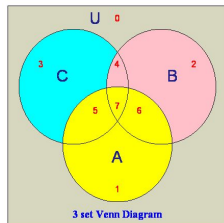
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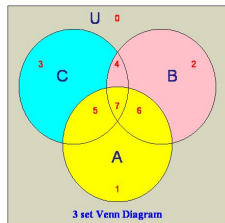
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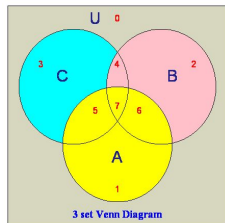
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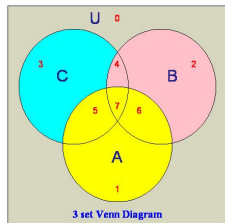
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- We can make a three set counting formula using the Venn diagram as before.
- The formula is: $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$.
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- So, for four or more sets, one has to learn how to use the set builder notation carefully and resolve set operations.
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- So, for four or more sets, one has to learn how to use the set builder notation carefully and resolve set operations.
- As a result, some people prefer to do the two and three set calculations also using formulas and set builder notation.
- The two basic counting formulas that one should remember are:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

and

$$\begin{aligned} n(A \cup B \cup C) = & n(A) + n(B) + n(C) \\ & - (n(A \cap B) + n(B \cap C) + n(C \cap A)) . \\ & + n(A \cap B \cap C) \end{aligned}$$