DEPARTMENT OF MATHEMATICS

Ma162 Samples from old Final Exams

This is for your practice.

1. Fred Foy has $\$100,000$ to invest in stocks, bonds and a money market account. The stocks have an expected return rate of 10% per year, the bonds pay 6% per year and the money market earns 2% per year.

   Fred insists that the money in the money market funds should equal the sum of 20% of the amount invested in stocks and 10% of the amount invested in bonds.

   Fred wants to allocate the $\$100,000$ among the three investments in order to provide an expected income of $\$8,000$ per year.

   Use x, y, z to denote the amounts invested in stocks, bonds and money market respectively.

   Answer the following questions.

   (a) Write down a linear equation in x, y, z relating to the total investment.

   (b) Write down a linear equation relating to the expected annual income.

   (c) Write down additional linear equation(s) as needed to express the remaining conditions on the investments.

   (d) Construct an appropriate augmented matrix to solve all the linear equations constructed above. It is not necessary to operate on this matrix any further at this point.

   (e) Solve the linear equations represented by the following augmented matrix.

$$\left[\begin{array}{rrr|r} x & y & z & RHS \\ 1 & 1 & 1 & 100 \\ 5 & 3 & 1 & 400 \\ 2 & 1 & -10 & 0 \end{array}\right]$$

2. Perth mining company operates two mines for the purpose of extracting gold and silver.

   The Saddle mine costs $\$14,000$ per day to operate and it yields 50 ounces of gold and 3,000 ounces of silver each day. The Horseshoe mine costs $\$16,000$ per day to operate and yields 75 ounces of gold and 1,000 ounces of silver each day.

   The company management has a target of at least 650 ounces of gold and at least 18,000 ounces of silver.

   Set up a linear programming problem whose solution will determine how many days each mine should be operated to reach the target while minimizing the cost of operation.

   You must define all the variables and list all the constraints. It is not necessary to solve the problem.

3. Consider these inequalities and answer the questions below.

$$x + y \leq 6 \ , \ 2x + y \leq 8 \ , \ x \geq 0 \ , \ y \geq 0.$$

(a) Graph the feasible set of the above inequalities on the given graph paper.

(b) List all the corner points of the feasible set. Be sure to mark them on your graph as well.

(c) Determine the maximum value of $2x + 6y$ on the above feasible set. Evaluation of $2x + 6y$ produces maximum value 36 at $(0, 6)$.

4. In a group of 110 people, it is determined that 52 drink coffee, 41 drink tea and 39 don't drink either. A person is selected at random.

   Answer the following:

   (a) Compute the probability that the selected person drinks at least one of coffee or tea.

   (b) Compute the probability that the selected person drinks both coffee and tea.

   (c) Compute the probability that the selected person drinks only coffee.

   (d) In this experiment of selecting one person at random, are the events "is a coffee drinker" and "is a tea drinker" independent? Be sure to explain your answer.

5. A bin in the hi-fi department of a bargain outlet contains 120 cassette tapes of which 15 are known to be defective. A customer randomly selects 7 of the tapes.

   Answer the following:

   (a) Describe the sample space and determine the number of elements in the sample space.

   (b) Find the probability that exactly 2 of the chosen tapes are defective.

   (c) Find the probability that at least 1 of the chosen tapes is defective.

   (d) Find the probability that none of the chosen tapes are defective.

6. In a high school there are 400 seniors of which 250 are female. 70% of the females and 50% of the males have their driver's licences.

   A student is chosen at random from the senior class.

   Answer the following:

   (a) What is the probability that the chosen student is a female with a driver's licence?

   (b) What is the probability that the chosen student is a male with a driver's licence?

   (c) What is the probability that the chosen student has a driver's licence?

   (d) Given that the chosen student does not have a driver's licence, what is the probability that the student is a male?

7. Set this problem up, by stating the chosen variables, the function to be maximized and **all** the inequalities. **Do not solve the problem.**

WidgetSS makes and sells pulleys and sprockets.

Each pulley sells for $ 18, requires 1.5 hours of finishing and 3.5 hours of machining.

Each sprocket sells for $ 20, requires 2.5 hours of finishing and 1.5 hours of machining.

The company has 100 hours of finishing time and 90 hours of machining time available.

Set up a LPP whose solution will determine how many pulleys and how many sprockets should be made to maximize the company profit.

i) Define and explain all the variables you use.

ii) Now describe the LPP explicitly. We have the objective function $P = 18x + 20y$ to be maximized. The conditions are:

iii) The initial Simplex tableau is:

8. i) Sketch and shade the region described by the inequalities. Compute the coordinates of the corner points and mark them on your graph.

$x + y \leq 5$

$y \geq 3$

$x \geq 0$, $y \geq 0$

ii) Find the maximum value of the function, $P = 3\,x + 5\,y$ on the region.

9. Here is a final tableau associated with a maximal LPP.

x	y	z	s	t	u	P	constants
-2	0	1	0	-4	-13	0	6
-1	1	0	0	0	1	0	4
1	0	0	1	1	4	0	2
2	0	0	0	1	9	1	7

Using your knowledge of the Simplex algorithm, determine the solution to the maximal LPP.

If we were to read the solution of the dual minimization problem from the same tableau, then its variables would correspond to s, t, u with values $0, 1, 9$.

Thus the minimization problem solution is:

10. Set this problem up, by stating the chosen variables, the equations to be solved and **the initial augmented matrix. Do not solve the problem.**

A toymaking company "Plaything" has a contract to supply 340 toys. It can supply any combination of three different types of toys with model names A,B,C. The cost of materials for the three types of toys is \$40,\$50, \$60 respectively.

Plaything has a total of $\$\ 17,400$. available for buying the materials.

Each toy of type A requires 40 labor-hours, each toy of type B requires 25 labor-hours and each toy of type C requires 20 labor-hours.

Plaything has a total of $9,300$. labor-hours available. Plaything wants to decide how many toys of each type should be manufactured to use all the resources and also fulfill the contract.

Use x,y,z to denote the number of toys produced of each of the types A,B,C respectively.

The equations to be solved are:

The augmented matrix is:

11. (i) Consider the following system of linear equations.

$$x + 2\,y + z = 2$$

$$2\,x + 5\,y + 3\,z = 7$$

Write down the augmented matrix for this system of equations.

Reduce the augmented matrix to REF (the row echelon form). It is essential to show the steps of row reductions and explicitly write the row operations used.

(ii) Using above calculations, determine all the solutions to the system of equations in x, y, z given above.

12. A group of 153 people were asked if they drink Coke or Pepsi.

It was found that 68 people drink Coke and 78 people drink Pepsi, while 25 people announced that they drink no soft drink!

(i) Estimate the probability that a random person drinks at least one of the two soft drinks.

(ii) Estimate the probability that a random person drinks both Coke and Pepsi.

(iii) Estimate the probability that a random person drinks only Pepsi.

13. An experiment consists of casting a die and observing the number on top. It is found that 54% of the time the number on top is 1 or 3 or 5 or 6.

It is also observed for the same die that 63% of the time the number on top less than or equal to 4. Answer the following questions **based on these observations.**

(i) What is the probability that the number on top is 1 or 3 ?

(ii) What is the probability that the number on top is 2 or 4 ?

(iii) The experimenter concludes that this die must be loaded since a fair die would show a number less than or equal to 4 on top for ____% of the time.

14. Two fair dice are tossed, one red and one blue.

Let A be the event that the sum of the numbers on top is 7.

Let B be the event that the number on top of the red die is **not** 2. Answer the following:

(i) What is the probability that the sum of the numbers on top is 7, i.e. what is $P(A)$?

(ii) What is the probability that the sum of the numbers on top is 7, given that the number on top of the red die is **not** 2, i.e. what is $P(A|B)$?

(iii) Are the events A and B independent? Explain your reasoning.

Suggestion. Be sure to keep at least four digits of accuracy.