DEPARTMENT OF MATHEMATICS

Ma213 - EXAM #3 Fall 2012

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO

There are 7 problems and a total of 8 pages including this one.

Problem	**Maximum Score**	**Actual Score**
1	20	
2	10	
3	10	
4	10	
5	10	
6	15	
7	15	
WHS	10	
Total	100	

NAME: __________________________________

SECTION No: __________________________________

LAST 4 digits of Student No.: __________________________________

1. **20 points**

(a) Let $f(x,y) = x^2 + 3\,xy + 5\,y^2 + 4\,x^3 - y^3$. Use the second derivative test to decide if the point $(0,0)$ is a local minimum or local maximum or a saddle point. If the test is inconclusive, explain why.

Grading Policy: 2 pts for D, 2 pts for finish. **Answer:** $D = 11$, $f_{xx}(0,0) > 0$, so local min.

(b) Let $f(x,y) = x^2 + 3\,xy - 5\,y^2 + x^3 - 3\,y^3$. Use the second derivative test to decide if the point $(0,0)$ is a local minimum or local maximum or a saddle point. If the test is inconclusive, explain why.

Grading Policy: 2 pts for D, 2 pts for finish. **Answer:** $D = -29$, so saddle point.

(c) Let $f(x,y) = -2\,x^2 + 7\,xy - 10\,y^2 + x^3 - 3\,y^3$. Use the second derivative test to decide if the point $(0,0)$ is a local minimum or local maximum or a saddle point. If the test is inconclusive, explain why.

Grading Policy: 2 pts for D, 2 pts for finish. **Answer:** $D = 31$, $f_{xx}(0,0) < 0$, so local max.

(d) Use Lagrange Multipliers to calculate the **minimum** value of the function $x^2 + y^2 + z^2$ on the surface $2x + y + 2z = 9$.

Grading Policy: 2pts. setup, 4 pts. to find the point, 2 pts. for the finish. **Answer:** Set $F = x^2 + y^2 + z^2 - \lambda(2x + y + 2z - 9)$.

Using F_x, F_y, F_z deduce $(x,y,z) = \lambda(1, 1/2, 1)$.

Plugging into $2x + y + 2z = 9$ get $\lambda = 2$.

The minimum point is $(2,1,2)$ and min. distance is 9.

2. **10 points**

(a) Sketch the region of integration for the following integral

$$\int_0^4 \int_{-\sqrt{4-y}}^{\sqrt{4-y}} dx\, dy.$$

Grading Policy: 3 pts. **Answer:** The area between the parabola $y = 4 - x^2$ and the x-axis.

(b) Now reverse the order of integration for the above integral and write out the new limits. **Grading Policy:** 3 pts. No credit if the order is not reversed. **Answer:**

$$\int_{x=-2}^{2} \int_{y=0}^{4-x^2} dy\, dx.$$

(c) Finally determine the mass of a lamina in the shape of the above region of integration if it has a uniform density 3.

Grading Policy: 4 pts. **Answer:** Evaluates to 32.

3. **10 points**

Convert the given integral to polar coordinates and then evaluate it.

$$\int\int_D \sqrt{(x^2 + y^2 - 1)} dA$$

Here D is the region between circles of radius $1, 2$ centered at the origin.

Grading Policy: 4pts. for correct limits, 2 pts. for conversion of differential, 4 pts. to finish integration. **Answer:**

$$\int_{\theta=0}^{2\pi} \int_{r=1}^{2} \sqrt{r^2 - 1} r \, dr \, d\theta = 2\sqrt{3}\pi.$$

4. **10 points**

The probability density function $f(x, y)$ for a pair of random variables is given by

$$f(x,y) = \begin{cases} xy/4 & \text{if the point } (x,y) \text{ lies in } \text{ the square } 0 \leq x, y \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

Use this function to find the probability that the variables lie in the region given by

$$x \geq 0, \quad y \geq 0, \quad x + y \leq 1.$$

Be sure to sketch the region before you proceed.

Grading Policy: 3pts. for the correct region. 7 pts for the correct integral. **Answer:** The region is a triangle with corners $(0,0), (1,0), (0,1)$. The integral

$$\int_{x=0}^{1} \int_{y=0}^{1-x} \frac{xy}{4} \, dy \, dx = 1/96.$$

5. **10 points**

Evaluate the integral:

$$\int_0^1 \int_{y^2}^{y} y dx\, dy$$

Sketch the region of integration and give at least one interpretation for the value of the integral in terms of the region of integration. Think where you might have seen such an integral in applications.

Grading Policy: 6 pts. to evaluate. 4 pts. for sketch and a meaningful interpretation. **Answer:** The value is $1/12$. The region between the parabola $x = y^2$ and line $x = y$. It evaluates the moment about x-axis. It is also $1/6$ times the y-coordinate of the center of mass of the region.

6. **15 points**

The formula for the surface area of a surface which is given parametrically by:

$$\mathbf{r}(u,v) =< x(u,v), y(u,v), z(u,v) >$$

where (u,v) are in a region R is:

$$\int\int_R |\mathbf{r}_u \times \mathbf{r}_v| dA$$

Grading Policy: 5 points for the cross product. 4 pts. for correct integrand. 6 pts. to finish.

(a) Suppose that a parametric surface is given by

$$\mathbf{r}(u,v) =< u, v, uv > .$$

Calculate

$$\mathbf{r}_u \times \mathbf{r}_v .$$

Answer: $< 1, 0, v > \times < 0, 1, u > =< -v, -u, 1 >$.

(b) Find the area of the part of the above surface where (u,v) are in the unit circle $u^2 + v^2 = 1$.

Hint: Polar coordinates are good!

Answer: The integral evaluates to $2\left(-1/3 + 2/3\sqrt{2}\right)\pi$.

7. **15 points** Sketch the region of integration for the triple integral

$$\int_{x=0}^{4} \int_{z=0}^{\sqrt{16-x^2}} \int_{y=0}^{x} yz dy\, dz\, dx$$

Grading Policy: 9+6 pts..

(a) Evaluate the integral.
Answer: 512/15.

(b) Calculate the limits for the same integral when you wish to change the order to $dz\, dy\, dx$.
Answer: The region has xy projection as a triangle with vertices $(0,0), (4,0), (4,4)$. So the integral is

$$\int_{x=0}^{4} \int_{y=0}^{x} \int_{z=0}^{\sqrt{16-x^2}} yz dz\, dy\, dx.$$