Formulas for Applications.

1. **Mass**

   $$m = \int\int_R \rho(x,y)dA \text{ for plane region } R \text{ and } \int\int\int_R \rho(x,y,z)dv \text{ for solid region } R$$

   where ρ is the density function.

2. **Moment** The moment about y axis is $M_y = \int\int_R x\rho(x,y)dA$.

   Moment about x axis is $M_x = \int\int_R y\rho(x,y)dA$.

   More generally, for moment about a general line, use signed distance from it.

   There is a similar extension for three dimensions.

3. **Center of mass**

   Center of mass is

   $$(\overline{x},\overline{y}) = (\frac{M_y}{m}, \frac{M_x}{m})$$

   There is a similar extension for three dimensions.

4. **Second Moment of Moment of Inertia**

   Second moment about y axis is $I_y = \int\int_R x^2\rho(x,y)dA$.

   Second moment about x axis is $I_x = \int\int_R y^2\rho(x,y)dA$.

   More generally, for Second moment about a general line, use square of the distance from it.

5. **Radius of gyration**

   The radius of gyration about x-axis is defined by $\sqrt{\frac{I_x}{m}}$.

   The radius of gyration about y-axis is defined by $\sqrt{\frac{I_y}{m}}$.

   The radius of gyration about a general line is defined similarly.

   The radius of gyration about the origin is $\int\int_R (x^2+y^2)\rho(x,y)dA$.

   Corresponding radius about a point (a,b) uses $(x-a)^2+(y-b)^2$ in place of x^2+y^2.

6. **Probability**

   Let $p(x,y)$ be the probability density function.

   Then probability of a point being in a region R is $\int\int_R p(x,y)dA$.

   The expected value of a function $f(x,y)$ on R is $\int\int_R f(x,y)p(x,y)dA$.