Spring 2018 General Information.

The examination will have 6 questions. You should study using the homework problems, quizzes, class notes as well as online notes and your own work.

Generally, you should simplify answers and do correct algebraic modifications. Often, it is OK to leave radicals and other calculator functions like arccos unevaluated, unless the question asks for simplification. You are expected to know the basic values and standard identities from trigonometry.

Thus, when I have provided decimal answers below, the only reason is not to give away the intermediate steps and thus force you to carry out the steps yourself and verify the final answer.

1. Given vectors $v =< 1, -3, -4 >, w =< 4, 3, 8 >, h =< 2, 1, 3 >$ calculate the following:

   (a) $|v|$. **Answer:**

   (b) Find $\angle(v, w)$**Answer:**

   (c) Find c such that $v + cw$ is perpendicular to h. **Answer:**

   (d) Find all vector perpendicular to v, w and having length 30. **Answer:**

   (e) Find equations for a line passing through $(1, 1, 1)$ and perpendicular to the plane containing vectors parallel to v, w. **Answer:**

   (f) Find the equation of a plane passing through $(1, 1, 1)$ and containing vectors parallel to v, w. **Answer:**

   (g) Find $Proj_w v$. Also find the scalar projection of v on w. **Answer:**

   (h) Find the volume of the parallelepiped formed by the three vectors. **Answer:**

   (i) Calculate the area of the parallelogram formed with sides v, w. **Answer:**

2. Find the equation of a plane through $(1, 2, 3), (2, 3, 4), (4, 3, 1)$. **Answer:**

3. Find the equation of a line passing through $(1, 1, 1)$ perpendicular to the plane above. **Answer:**

4. Find the distance of $(1, 1, 1)$ from the above plane.**Answer:**

5. Find the distance between the lines

$$\mathbf{r}(t) =< 1 + t, 2 - t, 3 + 2t > \text{ and } \mathbf{r}(s) =< 2 - s, 3 - s, 5 + s > .$$

   Answer:

6. Given $|u| = 3$ and $|v| = 5$ with $u \cdot v = 10$, calculate $|u \times v|$. **Answer:**

7. Find the distance from $(1, 0, 2)$ to the line $\mathbf{r}(t) =< 1 + t, 2 - t, 3 + 2t >$. **Answer:**

8. Find the distance between the planes $2x + 3y - z = 5, 3x + 4y + 7z = 10$. **Answer:**

9. Find the distance between the planes $2x + 3y - z = 5, -4x - 6y + 2z = 1$. **Answer:**

10. What is the locus of all points which are equidistant from two points $(1, 1, 1)$ and $(2, 3, 5)$. **Answer:**

11. What is the locus of all points whose distance from $(1, 1, 1)$ is 4 times its distance to $(2, 3, 5)$. **Answer:**

12. Let a space curve be defined by $\mathbf{r}(t) =< \sin(2t), \cos(2t), t >$. Answer the following questions.

    (a) Calculate $\mathbf{r}'(t), \mathbf{r}''(t), \mathbf{r}'(t) \times \mathbf{r}''(t)$. **Answer:**

    (b) Find the equation(s) of the tangent line at $t = \pi/6$. **Answer:**

    (c) Calculate the arclength from $t = 0$ to $t = \pi/6$. **Answer:**

13. You should practice problems of conversion of coordinates and identification of quadrics by studying homework problems and notes.

14. Here are some problems on quadrics to try:

    (a) Give examples of: an ellipsoid, a hyperboloid of one sheet, an elliptic paraboloid, a cone with axis along the z-axis, a cylinder with axis parallel to the y-axis.

    (b) If $f(x, y, z) = 2z^2 + ax^2 + by^2 = c$, then different values of a, b, c will produce different examples. How many can you construct?

    (c) The surface $z = x^2 - y^2$ is cut by horizontal planes. What kind of conics arise? (Don't forget $z = 0$.) Answer a similar question for vertical planes of the type $x = a$ or $y = b$. Make brief sketches of distinct samples in each case.

    (d) Describe the nature of the surface $\rho = 4$.

    (e) Describe the nature of $\phi = \pi/4$.

    (f) Describe the nature of the surface $r = 4$.

    (g) Describe the nature of $\theta = \pi/4$.

    (h) What is the equation of $z = xy$ in cylindrical coordinates? In spherical coordinates?