General Information. **Subject to amendment**

The exam 2 will have 6 questions. You should study using WHS problems, quizzes, class notes as well as on line notes and your own work.

Generally, you should simplify answers and do correct algebraic modifications. You are expected to know the basic values from trigonometry and the standard identities. You must do simplifications when possible, but approximate decimal answers are not recommended. Thus, when I have provided decimal answers below, the only reason is not to give away the intermediate steps and thus force you to carry out the steps yourself and use the final answer only as a confirmation.

You are permitted to bring a two sided sheet of notes of your own, as before.

1. Given position vector for a particle in space $P(t) =< t^2 + 5t, -t^2 + 3t, 2t + 3 >$, calculate the following quantities. Generally, it is OK to leave radicals unevaluated, but it is good to simplify expressions so that later calculations are manageable.

   (a) Velocity vector v and speed $|v|$. **Answer:**

   (b) Find acceleration a. **Answer:**

   (c) Find B. **Answer:**

   (d) Find unit normal N. **Answer:**

   (e) Calculate T, B, N at $t = 0$. **Answer:**

   (f) Calculate the curvature at $t = 0$. **Answer:**

   (g) Is this a plane curve? Explain. **Answer:**

   (h) Find the tangential and normal components of a at $t = 0$, **Answer:**

2. Find where the function $\frac{x^3+y^3+xy}{x(x+y)(x^4+y^4)}$ is continuous. **Answer:**

3. Find where $\log(\frac{1-xy}{1+x^2y^2})$ is continuous. **Answer:**

4. What are the level curves of the function $z = x^2 + y^2 + 2x + 6y$. Deduce the range of the function.**Answer:**

5. Find the limit as $(x, y) \to (0, 0)$ if it exists. If it does not, then explain why. $f(x, y) = \log(\frac{x^2+y^2+4}{2x+3y+5})$. **Answer:**

6. Find the limit as $(x, y) \to (0, 0)$ if it exists. If it does not, then explain why. $f(x, y) = \frac{y^4}{x^4+y^4}$. **Answer:**

7. Find the limit as $(x, y) \to (0, 0)$ if it exists. If it does not, then explain why. $f(x, y) = \frac{xy^4}{x^2+y^2}$. **Answer:**

8. Find the limit as $(x, y) \to (0, 0)$ if it exists. If it does not, then explain why. $f(x, y) = \frac{xy^2}{x^2+y^4}$. **Answer:**

9. Let

   $f(x, y) = x^3-3y^3+xy+5x, x = g(u, v) = 3uv+u+v, y = h(u, v) = u^2-2v^2, P(x, y, z) = x^3+y^2+xyz.$

   Answer the following questions.

   (a) Find $\nabla(f), \nabla(g), \nabla(h)$. **Answer:**

   (b) Find $\frac{\partial f}{\partial u}$. **Answer:**

   (c) Find the value of $\frac{\partial f}{\partial u}$ when $u = 2, v = -1$. **Answer:**

   (d) Calculate the Laplacian of each of these functions. **Answer:**

   (e) What is $D_u(P)(1, -2, -3)$ when $u =< 1, -2, 1 >$? Which directions give the smallest and the largest directional derivative? Which direction gives zero derivative? **Answer:**

10. If $x^2 + y^3 + z^5 + 6xyz = 9$, then find $\frac{\partial z}{\partial x}$. Also evaluate it when $x = y = z = 1$. **Answer:**

11. Volume of a pyramid is 1/3 times the area of its base times its height. If $f(x, y)$ is the volume of a pyramid whose base is a square of side x and has height y, use differentials to find a linear approximation function for the volume when $x = 35$ and $y = 15$. Use this to estimate the volume of a pyramid whose base has side 36 and height 14. **Answer:**

12. What is the equation of the tangent plane to $x^2 + y^2 - z^2 + xy + x + z + 8 = 0$. Find the tangent plane to it at $(x, y, z) = (1, -2, -3)$. Using the tangent plane approximation, **estimate** c if $(1.2, c, -2.8)$ is on the surface. **Answer:**

13. Find all the critical points of these functions.

    $f(x, y) = x^2+4xy+y^2-2x-3y, g(x, y) = x^2-y^3+3\,y+x, h(x, y, z) = x^2-3\,y^2+z^2+xy+yz-7\,z.$

    Answer:

14. Testing for max/min. Consider the function $f(x, y) = 3x^2 + axy + 4y^2 + x^2 * y^2$ where a is a constant. Show that $f(x, y)$ has a critical point at $(0, 0)$ and determine its nature for various values of a. Note that four answers are possible: 1. Local max. 2. Local min. 3. Saddle point. 4. Test is inconclusive.

    Answer:

15. Consider similar problems by taking parameters like a, b etc. in different places. For example, analyze $2x^2 + 3axy + (4 - a)y^2$.

    Also note that terms of degree bigger than 2 can be inserted at will, when you are analyzing the function at $(0, 0)$.

    Answer: