

### General Information.

The examination **will have 6 questions**. You should study using WHS problems, quizzes, class notes as well as on line notes and your own work.

Generally, you should simplify answers and do correct algebraic modifications. Often, it is OK to leave radicals and other calculator functions like arccos unevaluated. You are expected to know the basic values from trigonometry and the standard identities. You must do simplifications when possible, but approximate decimal answers are not recommended. Thus, when I have provided decimal answers below, the only reason is not to give away the intermediate steps and thus force you to carry out the steps yourself and use the final answer only as a confirmation.

You are permitted to bring your own two sided sheet of notes, as before. No other material may be used during the exam.

1. Find all the critical points of the following functions and use the second derivative test to determine if it is an extremum point and if so, of what type. If the test is inconclusive, explain why.

(a)  $f(x, y) = x^3 + y^3 - 3xy + 4$ . **Answer:**

(b)  $f(x, y) = x^2 + 2xy + y^2 + x^3$ . **Answer:**

(c)  $f(x, y) = e^{4y-x^2+y^2}$ . **Answer:**

2. Use Lagrange Multipliers to solve the following problems.

(a) Find the maximum volume of a rectangular box with  $(0, 0, 0)$  and  $(x, y, z)$  as opposite corners where  $x, y, z$  satisfy  $5x + 4y + 5z = 60$ . **Answer:**

(b) Find the minimum surface area of a box without a top of volume 500 cubic inches. **Answer:**

(c) Find the distance of the origin from the surface  $xyz = 1000$ . **Answer:**

3. Sketch the area of integration of the given integral and reverse the order of integration. Evaluate the integral.

(a)  $\int_{x=0}^1 \int_{y=1-\sqrt{1-x}}^{1+\sqrt{1-x}} dy dx$ . Also, evaluate the mass if the density function is  $1 + x$ .  
**Answer:**

(b)  $\int_{y=0}^5 \int_{x=-y/5}^{y/5} dx dy$ .

Also, evaluate the mass and the center of mass if the density function is  $y^2$ .

**Answer:**

4. Calculate the indicated integrals. Use polar/cylindrical/spherical coordinates as appropriate, or other convenient changes of coordinates. Do practice writing limits in the original coordinates.

(a)  $\iiint_R \sqrt{x^2 + y^2} \, dv$  where  $R$  is the region inside the sphere  $x^2 + y^2 + z^2 = 9$  and the first octant.

**Answer:**

(b)  $\iiint_R x \, dv$  where  $R$  is the region bounded by  $x^2 + y^2 = 1$ ,  $z = 0$  and  $z^2 = 4(x^2 + y^2)$ .

**Answer:**

**Note:** Consider changing the  $x$  to  $y$ ,  $z$  and 1 respectively. These calculations will let us compute the mass and center of mass, assuming a constant density function 1.

Verify:

$$m = \iiint_R 1 \, dv = 4\pi/3, m\bar{x} = \iiint_R x \, dv = 0, m\bar{y} = \iiint_R y \, dv = 0, m\bar{z} = \iiint_R z \, dv = \pi.$$

Thus, the center of mass is  $(0, 0, \frac{\pi}{4\pi/3}) = (0, 0, \frac{3}{4})$ . Of these three numbers, the first two zeros could be guessed from symmetry.

(c)  $\iiint_R x \, dv$  where  $R$  is the region bounded by  $x^2 + y^2 = 1$ ,  $z^2 = 4(x^2 + y^2)$  and lying in the first octant.

**Answer:**

(d)  $\iint_R (x + y) \, dx \, dy$  where  $R$  is the parallelogram with vertices  $(0, 0)$ ,  $(4, 5)$ ,  $(2, 3)$ ,  $(6, 8)$ . Do this by transforming the region  $R$  to the unit square  $S$  with corners  $(0, 0)$ ,  $(1, 0)$ ,  $(0, 1)$ ,  $(1, 1)$ .

**Answer:**

5. Additional formulas on surface area would be posted in a separate file soon.