DEPARTMENT OF MATHEMATICS

Ma213 - EXAM #1 Spring 2013 **Key**

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO

There are 7 problems and a total of 6 pages including this one. You are permitted **one** letter size sheet of notes (front and back) of your own notes. These will not be shared. Calculators are permitted, so long as they are not capable of wireless communication.

Problem	**Maximum Score**	**Earned Score**
1	20	
2	10	
3	10	
4	10	
5	10	
6	15	
7	15	
WHS	10	
Total	100	

NAME: ______________________________

SECTION No: ______________________________

LAST 4 digits of Student No.: ______________________________

1. (20 points)

Grading Policy: 4 points each for the 5 parts. Generally, one point off for minor mistakes, more for serious ones. Decimal evaluation is not needed, but wrong evaluation causes point loss.

(a) Consider the triangle formed by the points $P(1,3,2), Q(2,-1,3), R(1,2,-1)$

- Calculate the lengths of the sides PQ, PR.
  Answer: $3\sqrt{2}, \sqrt{10}$

- Calculate the angle at P.
  Answer: $\arccos(\frac{\sqrt{5}}{30})$ or $85.72546057°$.

- Calculate the area of the parallelogram formed by the sides PQ, PR.
  Answer: $PQ \times PR =< 13, 3, -1 >$. Hence the area is $\sqrt{13^2+3^2+1} = \sqrt{179}$.

(b) Find all vectors (in the plane) which are perpendicular to the vector $< -3, 17 >$ and **have length** 5.
Answer: $\pm \frac{5}{\sqrt{298}} < 17, 3 >$.

(c) Calculate **the scalar projection** of $-3\mathbf{i} + 17\mathbf{j}$ onto the vector $2\mathbf{i} - 2\mathbf{j}$.
Answer: $-10\sqrt{2}$.

2. (10 points)

   You are given vectors $v =< 3, -1, 5 >$ and $w =< -1, 2, 2 >$.

   Grading Policy: 5 points for each part.

   - Find a vector perpendicular to both v and w.
     Answer: $k < -12, -11, 5 >$.

   - Find the **volume of the parallelepiped** formed by the vectors v, w and $< 2, 1, -3 >$.
     Answer: $\begin{bmatrix} 3 & -1 & 5 \\ -1 & 2 & 2 \\ 2 & 1 & -3 \end{bmatrix}$ has determinant -50. So volume is 50.

3. (10 points) Consider the set of all points $P(x, y, z)$ such that the distance from $(1, 1, 1)$ to P is twice the distance from $(0, 0, 0)$ to P.

   Show that the set is a sphere.

   Find its center and radius.

   Grading Policy: 6 points for the correct equation. 2 points each for the center and the radius.
   Answer: The equation is: $4\,x^2 + 4\,y^2 + 4\,z^2 - (x-1)^2 - (y-1)^2 - (z-1)^2$. When simplified, it gives center $(-1/3, -1/3, -1/3)$ and radius $\sqrt{4/3}$.

4. (10 points)

Grading Policy: 6 points for the first part, 4 fr the second.

Find the equation(s) of the line L passing through $(3, -1, 5)$ and perpendicular to the plane $-x + 2\,y + 2\,z + 58 = 0$. Either symmetric equation(s) or the parametric form is acceptable.

Answer: $[x = 3 - t, y = -1 + 2\,t, z = 5 + 2\,t]$ or $\dfrac{x-3}{-1} = \dfrac{y+1}{2} = \dfrac{z-5}{2}$.

At what point does the line L meet the given plane?

Answer: $(10, -15, -9)$.

5. (10 points)

Grading Policy: For points 1+1+3, for plane 5. Find **three distinct points** P, Q, R on the line $\mathbf{r}(t) =< -2 + t, 2 - t, -1 + 3\,t >$ such that R is the midpoint of PQ.

Be sure to show your calculations or prove why your chosen points are on the line. Just guessed points will earn no credit.

Answer: Take any three values of t so that the third is the average of the first two. For example, $t = 0, 1, 1/2$.

Find the equation of the plane which passes through the point $(1, 1, 2)$ and contains the above line. *Hint: The plane must pass through at least two points on the line.*

Answer: $-10 + 6\,y + 2\,z = 0$, from $Det \begin{bmatrix} 2+x & -2+y & 1+z \\ 1 & -1 & 3 \\ 3 & -1 & 3 \end{bmatrix} = 0$.

Grading Policy: 3 points each. Lose one point if not simplified enough.

6. (15 points) A space curve is defined by the equations:

$$\mathbf{r}(t) =< cos(t), sin(t), 2t >$$

(a) Calculate the expressions $\mathbf{r}'(t)$, $\mathbf{r}''(t)$. **Simplify as much as possible.**

Answer: $< -\sin(t), \cos(t), 2 >< -\cos(t), -\sin(t), 0 >$

(b) Calculate $\mathbf{r}'(t) \times \mathbf{r}''(t)$. Simplify as much as possible.

Answer: $< 2\sin(t), -2\cos(t), 1 >$

(c) Find the equation(s) of the tangent line at $t = \pi/2$. Simplify as much as possible. Either parametric or symmetric form is acceptable.

Answer: $< x, y, z >=< -T, 1, \pi + 2T >$.

(d) Is there a value of t for which $\mathbf{r}'(t) =< 0, 0, 0 >$? Justify your answer.

Answer: Obviously no! The third component is 2!

(e) Find the arclength of the above space curve as t goes from 0 to $\frac{\pi}{2}$. It is important to set up the integral and then evaluate it.

Answer: $\int_0^{\pi/2} \sqrt{5}dt = \sqrt{(5)}\pi/2$.

Grading Policy: 5 points each. If expression is correct, calculator error is worth one point.

7. (15 points) Calculate the indicated quantities. You should write down the correct formulas, before evaluating. Use the calculator for the final answer.

   (a) Change the Cartesian coordinates $(3, 4, 1)$ to Cylindrical coordinates.

   Answer: Cylindrical $(5, \arctan(4/3), 1) = (5, 0.9272952179, 1)$. The angle in degrees would be $53.13010233°$.

   (b) Change the Cartesian coordinates $(3, 4, 1)$ to spherical coordinates.

   Answer: Spherical $(\sqrt{26}, \arctan(4/3), \arctan(5)) = (5.099019514, .9272952179, 1.373400767)$. The last two angles in degrees are:$53.13010234, 78.69006751$.

   (c) Change the Spherical coordinates $(\rho, \theta, \phi) = (4, \pi/6, \pi/6)$ to cartesian coordinates.

   Answer: $(\sqrt{3}, 1, 2\sqrt{3})$. Evaluation gives $1.732050808, 1, 3.464101616)$.