General Information.

The examination will have 7 questions. You should study using WHS problems, quizzes, class notes as well as on line notes and your own work.

Generally, you should simplify answers and do correct algebraic modifications. Often, it is OK to leave radicals and other calculator functions like arccos unevaluated. You are expected to know the basic values from trigonometry and the standard identities. You must do simplifications when possible, but approximate decimal answers are not recommended. Thus, when I have provided decimal answers below, the only reason is not to give away the intermediate steps and thus force you to carry out the steps yourself and use the final answer only as a confirmation.

You are permitted to bring one sheet of notes (front and back) of your own choice. No other books are notes are permitted.

1. Given vectors $v =< 1, -3, 4 >, w =< 4, 3, 7 >, h =< 2, 1, 3 >$ calculate the following:
   (a) $|v|$. **Answer:** $\sqrt{26}$
   (b) Find $\angle(v, w)$**Answer:** 1.018839036 radians or 58.37517676 degrees.
   (c) Find c such that $v + cw$ is perpendicular to h. **Answer:** $-\frac{11}{32}$.
   (d) Find all vector perpendicular to v, w and having length 3. **Answer:** $\pm\frac{1}{\sqrt{155}} < -33, 9, 15 >$.
   (e) Find equations for a line passing through $(1, 1, 1)$ and perpendicular to the plane containing vectors parallel to v, w. **Answer:** $x = 1 - 33t, y = 1 + 9t, z = 1 + 15t$
   (f) Find the equation of a plane passing through $(1, 1, 1)$ and containing vectors parallel to v, w. **Answer:** $-33(x - 1) + 9(y - 1) + 15(z - 1) = 0$.
   (g) Find $Proj_w v$. Also find the scalar projection of v on w. **Answer:** $\frac{23}{74} < 4, 3, 7 >$ and $\frac{23}{74}\sqrt{74}$.
   (h) Find the volume of the parallelepiped formed by the three vectors. **Answer:** Calculate the determinant whose rows are the three vectors. Take the absolute value. 12.
   (i) Calculate the area of the parallelogram formed with sides v, w. **Answer:** Calculate the length of $v \times w$. $\sqrt{33^2 + 9^2 + 15^2}$.
2. Find the equation of a plane through $(1, 2, 3), (2, 3, 4), (4, 3, 1)$. **Answer:** $-3\,x + 5\,y - 2\,z = 1$.
3. Find the equation of a line passing through $(1, 1, 1)$ perpendicular to the plane above. **Answer:** $x = 1 - 3t, y = 1 + 5t, z = 1 - 2t$.
4. Find the distance of $(1, 1, 1)$ from the above plane.**Answer:** 0.1622214211.
5. Find the distance between the lines

$$\mathbf{r}(t) =< 1 + t, 2 - t, 3 + 2t > \text{ and } \mathbf{r}(s) =< 2 - s, 3 - s, 5 + s > .$$

   Answer: 1.603567452.
6. Given $|u| = 3$ and $|v| = 5$ with $u \cdot v = 10$, calculate $|u \times v|$. **Answer:** $(3)(5)(\sqrt{1 - \frac{4}{9}}) = 11.18033988$.
7. Find the distance from $(1, 0, 2)$ to the line $\mathbf{r}(t) =< 1 + t, 2 - t, 3 + 2t >$. **Answer:** $\sqrt{5}$.

8. Find the distance between the planes $2x + 3y - z = 5, 3x + 4y + 7z = 10$. **Answer:** Since the normals to the planes are not proportional, the two planes must meet. The answer must be zero.

9. Find the distance between the planes $2x + 3y - z = 5, -4x - 6y + 2z = 1$. **Answer:** The planes have proportional normals, so they are parallel or identical. The distance is the same as the distance between any point of the first plane and the second plane. Take, for example, the point $(0, 0, -5)$ and find its distance to the second plane as $|-11|/\sqrt{16 + 36 + 4}$.

10. What is the locus of all points which are equidistant from two points $(1, 1, 1)$ and $(2, 3, 5)$. **Answer:** Equation is $(x-1)^2 + (y-1)^2 + (z-1)^2 = (x-2)^2 + (y-3)^2 + (z-5)^2$. Simplify to get a plane.

11. What is the locus of all points whose distance from $(1, 1, 1)$ is 4 times its distance to $(2, 3, 5)$. **Answer:** Equation is $(x-1)^2 + (y-1)^2 + (z-1)^2 = 4^2((x-2)^2 + (y-3)^2 + (z-5)^2)$. Simplify to get a sphere.

12. Let a space curve be defined by $\mathbf{r}(t) =< \sin(2t), \cos(2t), t >$. Answer the following questions.

    (a) Calculate $\mathbf{r}'(t), \mathbf{r}''(t), \mathbf{r}'(t) \times \mathbf{r}''(t)$. **Answer:**

    $$< 2\cos(2t), -2\sin(2t), 1 >, < -4\sin(2t), -4\cos(2t), 0 >, < 4\cos(2t), -4\sin(2t), -8 > .$$

    (b) Find the equation(s) of the tangent line at $t = \pi/6$. **Answer:**

    $$\mathbf{r}(t) =< \frac{\sqrt{3}}{2}.\frac{1}{2}, \frac{\pi}{6} > +t < 1, -\sqrt{3}, 1 > .$$

    (c) Calculate the arclength from $t = 0$ to $t = \pi/6$. **Answer:**

    $$\int_0^{\pi/6} \sqrt{4\cos^2(2t) + 4\sin^2(2t) + 1}\, dt = \sqrt{5}(\pi/6 - 0).$$

13. You should practice problems of conversion of coordinates and identification of quadrics by studying WHS problems and quizzes.