Quiz 14 Using Rank.

Ma322 Fall 2018 Avinash Sathaye

Suppose that A is a matrix such that the linear transformation $L(X) = AX$ maps vectors in $\Re^4$ to $\Re^3$.

Answer the following questions.

1. It is possible to decide if L is injective? State your conclusion with a brief explanation.

   Answer: A must be of type 3×4 and hence rank(A) is at most 3. The equations $AX = 0$ are consistent (being homogeneous) and would have at least one free variable, so there are non zero vectors X with $AX = 0$.

2. Suppose that rank(A) is known to be 2. Then you can deduce that L is not surjective.

   Prove this claim.

   Answer: The image of the map L is spanned by only two vectors and so cannot contain three independent vectors in $\Re^3$.

3. Write down an example of such an $A = A_{3\times 4}$ having rank 2. Then exhibit a concrete vector v not in the image of L.

   Hint: Try to write down a matrix A so that no row operations are needed for the conclusion!

   Answer: $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. It is clear that $v = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ cannot be of the form AX since the last row gives an inconsistent system.