Quiz 15 Vector spaces I.

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Let P_2 be the vector space consisting of all polynomials in one variable x with real coefficients and degree at most 2.

Answer the following questions.

- Prove that the set $B = \{1, x, x^2\}$ is a subset of P_2 with three independent vectors.

  Answer: By definition of polynomials, $a \cdot 1 + b \cdot x + c \cdot x^2 = 0$ iff $a = b = c = 0$.)

- Prove that B is a spanning set of P_2.

  Answer: Again by definition, any polynomial of degree at most two is of the form $a \cdot 1 + b \cdot x + c \cdot x^2$. So, they form a spanning set.

- Does it follow that $\dim(P_2) = 3$? Why?

  Answer: Now we know that B is a basis of P_2. Hence the number of elements in it, gives the dimension of P_2.

- For meditation: Let $f(x), g(x), h(x)$ be three polynomials in P_2. Construct a matrix using their coefficients and make tests to determine when $B = (f\ g\ h)$ is a basis of P_2.

  Answer: Write the coefficients as columns and get the 3×3 matrix. Now investigate the columns for the desired properties in $\Re^3$.