Quiz 16 Vector spaces II.

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Let $B = \begin{pmatrix} 1 & x & sin(x) \end{pmatrix}$ be the usual functions defined on $\Re$.
Let $V = \textbf{Span}(B)$. Answer the following questions.

1. Write down at least two members of V which are not in B.

   Answer: $1 - x$, $x + 2\sin(x)$, $\sin(x) + \pi$, etc.

2. Prove that B is a linearly independent set of vectors, V by following these steps.

   (a) Suppose that $a + bx + c\sin(x) = 0$ for some real numbers a, b, c. Write down the three equations in a, b, c by putting $x = 0, x = \pi/2, x = \pi$.
   Answer: $a + b(0) + c(0) = 0$, i.e. $a = 0$. $a + \pi/2b + c(1) = 0$ and $a + \pi b + c(0) = 0$.

   (b) Show that $a = b = c = 0$ is the only solution of the three equations.
   Answer: Using $a = 0$ the last equation gives $b = 0$. Using the second equation deduce $c = 0$.

   (c) Briefly explain why you are done! **Answer:** The above calculations show independence by definition.

3. Using the definition of a basis, show that B is a basis of V and hence V has dimension 3.

   Answer: Since the span of B is defined as V, B is its basis. Hence the dimension of V is 3.

4. For meditation: Verify that each member of B is a solution to $D^4(y) + D^2(y) = 0$ where $D = \frac{d}{dx}$ is the usual derivative w.r.t. x. The set W of all solutions of this equation is actually of dimension 4. Can you find a fourth element of its basis?

   Answer: $\cos(x)$ also satisfies the equation. How would you prove independence? There is another method also. Write down a linear combination with coefficients a, b, c, d. Since the resulting function