Quiz 17 Vector spaces coordinates.

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Suppose that V is a vector space over reals with a basis $B = \begin{pmatrix} v_1 & v_2 & v_3 & v_4 \end{pmatrix}$ Answer the following questions.

1. Explain why the four vectors in B are linearly independent and span V.

   Answer: This follows from the definition of a basis, which has exactly these two properties.

2. For each of the following vectors $w_i, i = 1 \cdots 4$ determine $[w_i]_B$.

$$w_1 = v_1, \;\; w_2 = 2v_2 + 3v_3, \;\; w_3 = v_1 + 2v_2 + 3v_3, \;\; w_4 = 2v_2 + 3v_3 + v_4.$$

   Answer: These are respectively columns of the matrix $M = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 2 & 2 \\ 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

3. Write $C = \begin{pmatrix} w_1 & w_2 & w_3 & w_4 \end{pmatrix} = BM$ for a 4×4 matrix M.

   Answer: M is the matrix of the above answer.

4. Determine if C is also a basis of V. Justify your answer.

   Answer: C will be a basis iff M is invertible. However, M has clearly dependent rows $2, 3$. Hence M has rank at most 3 and hence is singular. So the answer is no.

5. For meditation: Consider the polynomial $f(x) = 1 + x + x^2 + x^3 + x^4 + x^5$. If D denotes $\frac{d}{dx}$, explain why $f, D(f), D^2(f), \cdots, D^5(f)$ is a basis for P_5.

   More generally any six polynomials with distinct degrees from 0 to 5 is also a basis of P_5. This is easily extended to P_n for $n > 5$.

   Answer: Choose an order in the basis B such that the resulting matrix is visibly non singular.