Quiz 18 Change of Coordinates.

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1. If V and W are vector spaces over a field K, complete the following definition:

   A function $T : V \to W$ is said to be a linear transformation if:

   Answer: For all $v_1, v_2 \in V$ and $c \in K$, we have $L(v_1+v_2) = L(v_1)+L(v_2)$ and $L(cv) = cL(v)$.

2. Prove that the function $L : P_2 \to P_2$ defined by $L(p(x)) = p''(x)$ is a linear transformation.

   Answer: By rules of differentiation, we get $L(p+q) = (p+q)'' = p''+q'' = L(p) + L(q)$ and $L(cp) = (cp)'' = cp'' = cL(p)$.

3. Determine the $Ker(L) = \{p(x) \in P_2 \mid L(p(x)) = 0.\}$. Give a basis of the $Ker(L)$ and its dimension.

   Answer: We know that all polynomials with second derivative zero are linear polynomials, i.e. $p(x) = a + bx$ for $a, b \in \Re$. Thus $Ker(L) = Span\{1, x\}$. The basis is $(1, x)$ and dimension is 2.

4. Determine the $Image(L)$. Give a basis of $Image(L)$ and its dimension.

   Answer: If we start with a polynomial of degree at most 2, then its second derivative is a constant. For x^2, the second derivative is $2 \neq 0$. Hence $Image(L) = Span\{1\}$ and has dimension 1 with basis 1. Notice the relation: $\dim(P_2) = 3 = \dim(Ker(L)) + \dim(Image(L))$.

5. For meditation: Think of the Fundamental Theorem of Vector Spaces which says:

   Suppose that V is a finite dimensional space and we have a linear transformation $L : V \to W$ where W is some vector space. Then $\dim(V) = \dim(Ker(L)) + \dim(Image(L))$. Think why this might be true, at least in the usual euclidean spaces $\Re^n$.

   Answer: We will discuss this in class, of course.