Quiz 19 Change of Coordinates.

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Recall that a standard basis of $\Re^2$ is given by $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Let $v_1 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

Answer the following questions.

- Let $B = \begin{pmatrix} e_1 & e_2 \end{pmatrix}$ and $C = \begin{pmatrix} v_1 & v_2 \end{pmatrix}$. Determine the matrix P such that $C = BP$. Briefly explain why C is a basis of $\Re^2$.

  Useful comment: Recall that for any vector v, this gives the formula $P[v]_B = [v]_C$ and hence we can denote P as P_C^B to help us remember the formula.

  Answer: $\det(P) = \det(\begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix}) = -1 \neq 0$. So P has rank 2.

- Define a linear transformation $L : \Re^2 \to \Re^2$ by setting $L(v_1) = 5v_1$ and $L(v_2) = -5v_2$. Using linearity of L determine the formula for $L(av_1 + bv_2)$ as a combination of v_1, v_2, where a, b are arbitrary scalars.

  Hint: For example $L(2v_1 + 3v_2) = 2(5v_1) + 3(-5v_2) = 10v_1 - 15v_2$.

  Answer: $L(av_1 + bv_2) = 5av_1 - 5bv_2$.

- Use the above to determine $\mathcal{M}(L)_C^C = H$, the matrix identified by $[L(v)]_C = H[v]_C$. (*)This was not covered before the quiz, so dropped.

  Answer: $H = \begin{pmatrix} 5 & 0 \\ 0 & -5 \end{pmatrix}$.

- For meditation: The above formula for the matrix of transformation can be changed relative to the bases B, B by using the change of basis formula from the first question to yield: $P[L(v)]_C = PH[v]_C = PHP^{-1}P[v]_C$. This says that $L(v)_B = (PHP^{-1})[v]_B$. This shows how the matrix of transformation changes: namely $P\mathcal{M}(L)_C^C P^{-1} = \mathcal{M}(L)_B^B$. Often, we start with the more complicated $\mathcal{M}(L)_B^B$, since it is naturally given and try to find a suitable P to get a much more useful form H.