Quiz 21 Spanning sets.

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We are given that a vector space V over a field K is spanned by a vector sequence $H = \begin{pmatrix} u_1 & u_2 & u_3 & u_4 \end{pmatrix}$.

Answer the following questions.

1. What property should the vectors in H have if H is a basis for V?

   Answer: The vectors in H must be linearly independent.

   For all questions below, assume that H is a sequence of linearly independent vectors.

2. Determine if the vectors $B = \begin{pmatrix} u_1 + u_2 - u_3 & u_3 + u_4 & u_3 \end{pmatrix}$ are linearly independent. Justify your answer.

   Answer: Yes, they are. Their span is the same as the span of $\{u_1 + u_2, u_3, u_4\}$ and this is evidently an independent set. To prove this claim, note that u_3 is already in the span and since $u_3 + u_4$ is also in the span, it follows that u_4 is also in the span. Now from the first vector, we deduce that $u_1 + u_2$ is in the span.

3. Let $W = Span(B)$. Decide if $V = W$. Justify your answer. **Answer:** No! W has dimension $3 < 4 = \dim(V)$.

4. Find one vector v in V such v together with vectors in B also span V.

   Answer: We can take $v = u_1$ or $v = u_2$ or any combination of u_1, u_2 which is not a multiple of $u_1 + u_2$. Using v we can get both u_1, u_2 in our span and thus the span is V itself.

5. For meditation: Practice how a spanning set can be whittled down to a basis and an independent set can be enlarged to a basis. Think this out: If $P \subset Q$

are sets of vectors, then Q is independent implies P is independent. On the other hand, if $Q \subset Span(P)$, then $Span(Q) = Span(P)$.